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Got Local Food? Understanding the Fresh Produce Supply Chain

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Abstract

This paper studies which operational characteristics of the fresh produce supply chain help or hinder the viability of local food. We show how short lead time, constrained capacity, and the way prices are set in the fresh produce market create the local food paradox: the characteristics of local food that are appealing to the retailer induce supply chain dynamics that hurt the local farmer. A distinguishing characteristic of locally sourced fresh produce that has many operational implications is that it is grown closer to the end consumer than produce from the mainstream food supply chain. The local fresh produce supply is also often capacity constrained. Thus, the retailer cannot source only locally grown produce to satisfy demand. We find that the shorter lead time implied by local food proximity and the capacity constraint results in a retail order policy that is disadvantageous for the local farmer. The retailer's optimal order policy effectively uses the mainstream farm to satisfy the stable baseload demand and the local farm to react to demand volatility. However, unlike typical dual sourcing scenarios, the local farmer cannot capture any of the value created by his responsiveness because the produce price is set by the market. We study three mechanisms that can improve conditions for the local farm: coordination, backhauling, and a retail order policy, manager's discretion, that explicitly supports the local farmer. The operational mechanisms, coordination and backhauling, help the local farmer by making local supply more attractive to the retailer, inducing her to order more local produce. Coordination reduces the retailer's mismatch cost whereas backhauling increases the average margin. The manager's discretion order policy helps the local farm at the expense of retail profit. However, we show how combining manager's discretion with backhauling can benefit the retailer and local farmer. We also explore two ways in which sourcing local food can affect the environment: through the impact on food miles and by leveraging extended shelf-life to reduce retail food waste.

1 Introduction

The purpose of this paper is to determine which operational characteristics of the food supply chain help or hinder the viability of local food. In particular, we study the fresh produce supply chain. There are many dimensions associated with local food, including: 1) Proximity: food is grown a short distance from the end consumer 2) Quality: food is fresher and tastes better 3) Environment: fewer food miles are associated with local food and more environmentally friendly farming methods are used 4) Society: there are fair working conditions and compensation for farm workers.

In this paper, we focus on the proximity dimension because it is unequivocal and has many operational implications. The mainstream (industrial) food supply is typically concentrated in specific geographic regions. For example, two thirds of the acreage used to grow fresh tomatoes in the United

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States are in California or Florida (USDA 2012), and 60% of spinach is produced in California (AGMRC 2012). Thus, for the majority of consumers, fresh tomatoes from the mainstream supply travel a long distance in order to reach the end consumer. Although there is no precise distance definition for "local" food, common specifications used are "within 400 miles" or "within state." The difference in distance traveled affects the lead time of fresh produce supply. Moreover, mainstream farms are typically located in rural areas allowing farms to be large, whereas local farms are by definition close to urban areas where competition for land has forced farms to be smaller. Thus, the geographic characteristics of the fresh produce supply chain affect lead time and operational scale – two important operational parameters that we analyze in our model.

However, we acknowledge that quality, environment, and society are compelling arguments used by and for those interested in buying and promoting local food. Although every piece of local food is clearly not higher quality, better for the environment, and better for society than mainstream food, there is a general sense that food from local farms are more likely to exhibit these characteristics than mainstream food. In this paper, we do not take a position on whether local food is better than mainstream food. We take as given that there is increasing interest in growing the presence of local food in the food supply chain and investigate mechanisms for doing so.

We approach this by analyzing the operations of the fresh produce supply chain and showing how the structure and operating policies affect the presence of local food in our food supply. Using our understanding of how the fresh produce supply chain works, we identify and quantify the impact of operational changes that would increase the presence of local food in the fresh produce supply chain. The local farm that is the focus of our analysis is a mid-sized farm that grows for the wholesale market. The farm is too big to sell directly to consumers (e.g., through farmer's markets and community supported agriculture channels), but is not big enough to fill the entire demand for a large retail grocer.

Our model analyzes the optimal order policy of a grocery retailer who can order from a mainstream supplier that is located a far distance from the retail store and a local farm that is close. We derive the retailer's optimal order policy under two objective functions: maximizing profit and achieving a target service level. The local farm is capacity constrained and there is local supply uncertainty, but the lead time from the local farm to the retail store is significantly shorter than from the mainstream farm. We find that the retailer's optimal order policy is to order a constant quantity from the mainstream farmer and take advantage of the local farm's short lead time to react to demand volatility. This benefits the retailer by allowing her to reduce her mismatch cost. However, the local farmer sees a volatile order pattern, resulting in high overall risk for his business and also creating operational challenges such as staffing pick and pack lines. Unlike other dual sourcing situations where a highly responsive supplier can charge a premium for its service, fresh produce prices are determined by the market on the day the produce is sold. Thus, the local farmer has a significant disadvantage in the fresh produce supply chain: the pricing convention prevents the farmer from charging a premium for responsiveness. Our results show that there is a *local food paradox*: The proximity of the local food supply to the end consumer allows for shorter leadtime, thus providing an operational advantage for the retailer. However, we show that by exercising this advantage, the retailer places an operational burden on the local farm by using the local supplier to react to demand volatility.

We use our characterization of the fresh produce supply chain dynamics to investigate three mechanisms that could improve operating conditions for the local farmer: 1) coordination, 2) backhauling, and 3) manager's discretion order policy. An organization such as Red Tomato acts as a broker/distributor that aggregates produce from multiple local farms, effectively creating a larger, more reliable local supplier (redtomato.org, Alvarez et al. (2010)). Through coordination, Red Tomato can increase local capacity and decrease local supply uncertainty. This allows the retailer to better utilize the local farm to *reduce mismatch cost*. We show that this results in higher local order quantities, a more stable order pattern, and higher retail profit.

In some cases, the retailer may be able to reduce the transportation cost of locally sourced produce by leveraging backhauling. As trucks return empty from the retail store to the distribution center, they can be diverted to local farms to pick up fresh produce. Our analysis shows that the backhauling mechanism works very differently from coordination. Backhauling decreases the retailer's cost of ordering local produce and thus *increases her average margin*. The higher the fraction of produce the retailer sources from the local farmer, the higher her average margin. With backhauling, the retailer may actually incur higher mismatch cost by incorporating more uncertain local supply, but the increased mismatch cost is more than offset by higher average margin. Backhauling thus increases the quantity ordered from the local supplier, decreases local order volatility, increases local farm utilization, and increases retail profit.

We find, however, that if the retailer's objective is to achieve high service level rather than maximizing profit, coordination and backhauling are ineffective at improving the operating conditions for the local farmer. We investigate an order policy, which we call *manager's discretion*, that is used by Walmart to support local farms. Under the manager's discretion order policy, the retailer commits to buy everything the local farm produces. This clearly benefits the local farmer, but makes the retailer worse off. Under this order policy, the retailer loses the demand visibility advantage of the local farm's short lead time. Moreover, the retailer must incorporate local supply uncertainty into its safety stock when she orders from the mainstream supplier. Thus, retail profit decreases under manager's discretion. However, we show that if manager's discretion is combined with backhauling, if the unit ordering cost under backhauling is sufficiently low, the retailer can increase profit by sourcing locally and guarantee orders to the local farm. This is, in fact, the combination used by Walmart in its Heritage Agriculture Program.

We also explore how the proximity of local food to the consumer can affect the environmental impact of the food supply chain. The most direct application of our results is the effect on food miles. Although it is imperfect, food miles is often used as a measure of environmental sustainability. Using our results, the increase of local food as a percentage of the total supply can be quantified, and thus can be translated into a decrease in average food miles. Moreover, our results can provide a more nuanced interpretation of food miles by incorporating the effect of supply chain dynamics that change total food miles and food miles per unit of food consumed.

Another potential operational advantage afforded by the proximity of local food is the possibility of extending the shelf life of local food. If handled properly (i.e., if the produce enters the cold chain in a timely manner), produce from local farms can be harvested so that it has longer shelf life than produce from mainstream farms. Essentially, a portion of the transport time can be shifted to extend produce shelf life. This gives the retailer more opportunities to sell the produce and more flexibility in its order policy. We derive two heuristics for the retailer's order policy when local produce has extended shelf life. We show that appropriate handling to extend the shelf life of local food can increase the retailer's profit, improve the operating conditions for the local farm, and can also benefit the environment by reducing retail food waste.

Literature Review Our paper uses an operational lens to examine the viability of local food, a topic that has been studied in the food policy and agricultural economics literature (Abatekassa and Peterson (2011), Lerman (2012) provide overviews). Most of the extant work uses qualitative research methods, e.g., case studies, surveys, interviews to understand the factors that affect the viability of local food supply chains. King et al. (2011) provide an overview of issues involved in expanding the presence of local food, including distribution, packing, supply availability, and transportation. In a survey study by Heer and Mann (2010), they find that including different vertical players in the food supply chain is a critical factor for successful local food networks in Germany. A USDA report describes a number of cases studies that compare characteristics of local and mainstream food supply chains for apples, blueberries, leafy greens, beef, and milk (King et al. 2010). In a quantitative study, Nicholson et al. (2011) use a spatially-disaggregated transshipment model to determine the additional supply chain costs required for transportation and processing of dairy products if supply was localized. Our research complements this literature by explicitly analyzing how supply chain dynamics (i.e., inventory management decisions) change when local food is introduced as a sourcing option.

Our work builds on inventory theory; see Zipkin (2000) and Porteus (2002). In particular, it relates to the stream of literature that incorporates forecast updating into inventory control decisions, because the retailer can update her demand forecast between her ordering decisions from the mainstream and local farms. One important trade-off studied in this literature is between the improved demand forecast and the increased cost of ordering as one waits to acquire more information. Wang et al. (2012) studies this tradeoff and provides an overview of this literature; also see Ozer et al. (2007). This trade-off does not exist in our setting, because the local produce is not sold at a premium. In contrast, the key tension in our model is between improving the retailer's demand forecast and ensuring adequate supply, an issue that arises because the local supply is capacitated.

Another key aspect of our formulation is the perishability of fresh produce. Nahmias (2011) provides an overview of the perishable inventory literature; also see Karaesmen et al. (2011). This literature focuses primarily on dynamic models, which relate to our model of the extended shelf life of local produce (see Section 6). Characterization of the state-dependent optimal ordering policy for the two-period shelf life problem is provided by Nahmias and Pierskalla (1973) and Cohen (1976), both of which assume backlogging of unmet demand, and by Nahmias (1975) with a lost sales assumption.

These characterizations are implicit and their implementation using dynamic programming is difficult. We provide two simple heuristics to find the optimal ordering policy for the extended shelf life scenario. These heuristics do reasonably well in conveying the potential benefits of the longer shelf life to the retailer and the local farmer.

Lastly, our work relates to the literature on dual sourcing; see for example Johnson (2007), Veeraraghavan and Scheller-Wolf (2008) and Zhang et al. (2012). In dual sourcing, there is typically a low-cost, long leadtime supplier and a high-cost, short leadtime (responsive) supplier. The key notion in the dual sourcing literature is to source the base demand from the low-cost supplier, and source the excess demand from the expensive yet responsive supplier. The underlying tradeoff is different in our model than that in the dual sourcing literature, because the ordering cost is the same for both the local and mainstream suppliers. To be more specific, if the local farm were not capacity constrained, the retailer would only order from the local (responsive) supplier. Hence, the key tension in our model is due to the capacity constraint of the local supplier.

The paper is organized as follows. We present the model in Section 2. In Section 3 we derive the retailer's profit-maximizing order policy and show how coordination, backhauling, and manager's discretion affect the order policy and the operating conditions for the local farm. In Section 4, we derive the retailer's optimal order policy when her objective is to achieve a certain service level. We present a numerical example in Section 5. We discuss the environmental impact of the local food model in Section 6 and conclude in Section 7.

2 Model

At the most basic level, the fresh produce supply chain works like many others: the farmer (manufacturer) grows the produce, the retailer buys the produce from the farmer and sells it to the consumer. However, fresh produce is highly perishable, thus, we model the retailer's procurement strategy of a single fresh produce item which has a shelf life of one period.

We consider an infinite horizon problem assuming supply and demand are stationary. The retailer can supply its demand from a local farm or from a distant mainstream farm. We assume the lead time from the mainstream farm is two periods, and one period from the local farm. Adopting the terminology of Zipkin (2000), the timing of events is as follows. Define time t as the start of period t. To satisfy the demand in period t, the retailer orders y_m from the mainstream farm at time $t-1$ and orders y_l from the local farm at time t (see Figure 1). Thus, $y = y_l + y_m$ is the total amount ordered by the retailer to satisfy demand in period t.

Note that in order to have produce ready for sale in period t both the mainstream and local farms must plant crops months before period t. Therefore, the capacities of both sources are committed long before retail orders are received. The mainstream farm represents a supply source from a specialized agricultural region such as Florida, which produces more than two thirds of the tomatoes in the United States. Thus, we assume that the mainstream supplier is arbitrarily large and has enough capacity to satisfy any order from the retailer. Local farms that are close to urban areas are typically constrained by arable land availability (due to land development). For example, in a study of New York State, Peters et al. (2009) show that only 34% of the required food intake by New York residents can be supplied by local sources. Thus, we assume the local farm is capacity constrained. Moreover, local supply in each period is uncertain because the retailer lacks visibility on local supply or harvest yield is unpredictable. At time $t-1$, when the retailer orders from the mainstream farm, the local supply is uncertain. We model the local supply availability as $\mu + \epsilon_s$, where μ is the mean supply and ϵ_s is a normal, mean-zero random variable with variance σ_s^2. Local supply uncertainty is resolved prior to ordering from the local farm. We use $\tilde{\epsilon}_s$ to denote a realization of ϵ_s.

Because excess inventory perishes, it does not carry over from period t to period $t+1$. Therefore, the retailer starts each period with zero inventory and the quantity received at the beginning of each period is what is available for sale in that period. The resulting sales is the minimum of the demand and the inventory on-hand. We denote unit ordering cost and sales revenue for the retailer as w and r, respectively. These are the same for produce sourced from local and mainstream farms. Excess demand is lost. This infinite-horizon problem decomposes into a series of independent one-period problems.

Because the leadtime for ordering from the mainstream farm is longer than that from the local farm, the retailer faces higher demand uncertainty at the point she orders from the mainstream farm. We model demand uncertainty as follows. When ordering from the mainstream farm, the retailer's demand forecast is $\lambda + \epsilon_1 + \epsilon_2$, where λ is the mean demand, and ϵ_1 and ϵ_2 are normal, mean-zero

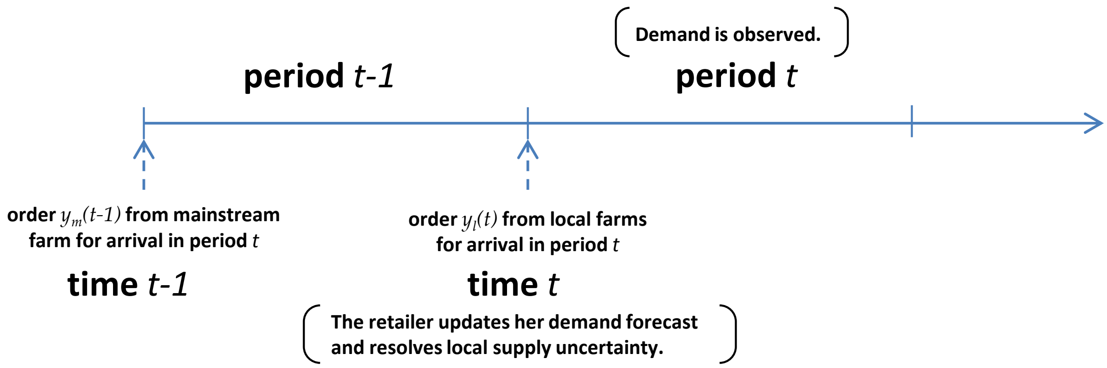

Figure 1: Timing of events.

random variables with variances given by σ_1^2 and σ_2^2, respectively. Prior to ordering from the local farm at time t, the retailer updates her demand forecast as follows: $\lambda + \tilde{\epsilon}_1 + \epsilon_2$, where $\tilde{\epsilon}_1$ denotes a realization of ϵ_1.

3 The Retailer's Profit-Maximizing Order Policy

In this section, we derive the optimal order policy of the retailer when her objective is to maximize profit. We will consider two sourcing policies. We first consider the straightforward case where the retailer only orders from one supplier, the mainstream farm (since the local farm does not have enough capacity to be the sole supplier). We call this the *mainstream only* sourcing policy. Then we will consider the *hybrid sourcing* policy, where the retailer sources from both the mainstream and local farms.

Under the mainstream only policy, maximizing profit is equivalent to minimizing the expected one period overage and underage costs. Thus, the retailer solves the following problem:

$$\min_{y \geq 0} w \, \mathbb{E}_{\epsilon_1,\epsilon_2}[(y - \lambda - \epsilon_1 - \epsilon_2)^+] + (r - w) \, \mathbb{E}_{\epsilon_1,\epsilon_2}[(\epsilon_1 + \epsilon_2 + \lambda - y)^+]. \tag{1}$$

This problem is a straightforward application of the newsvendor problem. The retailer's profit-maximizing order quantity is given by the following lemma.

Lemma 1 *Using the mainstream only sourcing policy, the retailer's profit-maximizing order quantity* *is*

$$y^* = \lambda + \sqrt{\sigma_1^2 + \sigma_2^2}\ \Phi^{-1}(\frac{r-w}{r}), \tag{2}$$

where $\Phi(\cdot)$ *denotes the cumulative distribution function of the standard normal distribution.*

If the retailer can choose to also source from the local farm (hybrid sourcing), she can do at least as well as when she sources only from the mainstream farm. Using backwards induction, we first consider the problem of how much to order from the local farm having ordered y_m from the mainstream farm. To minimize the expected one period cost of underage and overage, the retailer solves the following problem at time t

$$\min_{y \geq y_m} w\, \mathbb{E}_{\epsilon_2}[(y - \lambda - \tilde{\epsilon}_1 - \epsilon_2)^+] + (r-w)\, \mathbb{E}_{\epsilon_2}[(\lambda + \tilde{\epsilon}_1 + \epsilon_2 - y)^+]. \tag{3}$$

If we ignore the bound on y, this also is a typical newsvendor problem. The retailer's commitment to receive y_m units in period t from the mainstream farm constrains the newsvendor solution. That is, if the newsvendor solution is smaller than y_m for a particular period, the retailer does not order any produce from the local farm. The following proposition provides the optimal profit-maximizing quantity from the local farm, y_l^*. Note that y_l^* is the optimal order quantity, but the retailer actually receives $\min\{y_l^*, \mu + \tilde{\epsilon}_s\}$ from the local farm.

Proposition 1 *The unconstrained objective in Equation (3) is minimized at* $\tilde{y} = \lambda + \tilde{\epsilon}_1 + \sigma_2 \Phi^{-1}\left(\frac{r-w}{r}\right)$. *Thus, the retailer's profit-maximizing order quantity from the local farm is* $y_l^* = (\tilde{y} - y_m^*)^+$, *where* y_m^* *is the optimal order quantity from the mainstream farm and* $q^+ = \max(q, 0)$.

Next, we find the profit-maximizing order quantity from the mainstream farm, y_m^*. In choosing y_m^*, the retailer includes the capacity limit of the local farm in the optimization problem by constraining y from above by $y_m + \mu + \epsilon_s$ and solves the following for optimal y_m^*:

$$\min_{y_m \geq 0} \mathbb{E}_{\epsilon_1, \epsilon_s}\left[\min_{y_m + \mu + \epsilon_s \geq y \geq y_m} w\, \mathbb{E}_{\epsilon_2}[(y - \lambda - \epsilon_1 - \epsilon_2)^+] + (r-w)\, \mathbb{E}_{\epsilon_2}[(\lambda + \epsilon_1 + \epsilon_2 - y)^+]\right]. \tag{4}$$

The minimization between the square brackets in Equation (4) is the retailer's cost of overage plus cost of underage with the expectation taken over ϵ_2, the random component left at time t. The outer expectation is taken over ϵ_s and ϵ_1, both random at time $t-1$. The following proposition offers a characterization of the optimal y_m^*.

Proposition 2 *The optimal order quantity from the mainstream farm y_m^* solves the following:*

$$\int_{-\infty}^{(y_m-\sigma_2 z^*-\lambda)/\sigma_1} \big[1-\frac{r}{r-w}\,\Phi\big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\big)\big]\,\phi(u)du$$
$$+\Phi\big(\frac{-\mu}{\sigma_s}\big)\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty} \big[1-\frac{r}{r-w}\,\Phi\big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\big)\big]\,\phi(u)du$$
$$+\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty} \big[1-\frac{r}{r-w}\,\Phi\big(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2}\big)\big]\,\phi(u)du\phi(v)dv=0, \tag{5}$$

where $z^*=\Phi^{-1}\left(\frac{r-w}{r}\right)$.

From Proposition 2, we see that the profit-maximizing mainstream order y_m^* is determined by the model primitives. Thus, for any given set of parameters, y_m^* is fixed. However, Proposition 1 shows that the local order y_l^* depends on the realization of ϵ_1. Thus, a major implication of these results is that the retailer's profit-maximizing order strategy is to use local supply to react to demand volatility. By waiting to order from the local farm, the retailer sees a more accurate demand signal, and can adjust her order accordingly. The retailer essentially uses the local farm to capture demand upside. Thus, compared to when she follows the mainstream only sourcing policy, *the retailer's average mismatch cost (overage + underage) decreases if she also sources from the local farm (hybrid sourcing).*

However, the retailer's optimal order policy makes operating conditions very difficult at the local farm. The local farm sees a volatile order pattern – the local supply is used only when demand is higher than expected. The retailer reduces her risk of overstocking, but that burden is shifted to the local farmer because the farmer made the decision to plant months ago. The distance and resulting leadtime asymmetry causes *the local food paradox*: Shorter leadtime is what makes sourcing from local farms appealing to the retailer, but it is also what places local farmers at a disadvantage. The shorter leadtime of the local farm induces the retailer to use local sourcing as a mechanism to reduce overstocking by delaying the ordering of a portion of the supply to when demand is more certain.

3.1 Mechanisms that Improve Local Farm Operating Conditions

We now examine three mechanisms used in practice that can mitigate the adverse effects of the retailer's ordering strategy on the local farm: coordination, backhauling, and a retail store order policy which we will call "manager's discretion". In particular, we examine how these mechanisms increase the average amount sold by the local farm (thus better utilizing the capacity of the local farm), and how this affects the profit of the retailer. We derive analytical results in this section and in Section 5, we use these results in a numerical example to show the magnitude of the impact of each mechanism.

Coordination. The Red Tomato organization (redtomato.org, Alvarez et al. (2010)) acts as an intermediary between local farmers and retailers. Red Tomato facilitates communication between the retailer and local farms. In particular, sharing long-term forecasts enables farms to plant what the retailer anticipates selling. By coordinating the growing plan of local farms, it appears to the retailer that the local supply capacity increases. Moreover, by aggregating several farms, the pooling effect lowers the volatility of local supply. Local supply uncertainty can also be improved by communicating with the retailer what the local supply is several weeks before harvest, e.g., at time $t-1$.

The following two propositions show how increasing local capacity and decreasing local supply uncertainty affect the local farmer and the retailer.

Proposition 3 *As average local production capacity μ increases,*

1. *the average amount the retailer orders from the local farm increases,*
2. *the average amount sold by the local farm increases,*
3. *the retailer's expected one period cost decreases.*

Proposition 4 *As local supply uncertainty σ_s decreases,*

1. *the average amount the retailer orders from the local farm increases,*
2. *the average amount sold by the local farm increases,*
3. *the retailer's optimal expected one period cost decreases.*

By increasing local supply and decreasing local supply uncertainty, the retailer is able to manage her mismatch cost more effectively. Recall that the retailer uses the local farm to react to demand volatility. With a larger, more reliable local supply source, the retailer can delay a bigger portion of her order (i.e., order less from the mainstream supplier). Since a bigger portion of the retailer's order is delayed until the local order, the probability that demand materializes increases. Thus, the amount the retailer orders from the local farm increases. Since local supply is more reliable, the probability that the local farm can fill the order increases. At the same time, the retailer has better demand visibility on a bigger portion of her order so she can manage mismatch costs better.

Propositions 3 and 4 show that the Red Tomato model can benefit both the retailer and local farmer. As the amount of local supply increases or the local supply uncertainty decreases, the retailer purchases more from the local farm. Moreover, our numerical analysis in Section 5 shows that the local farmer sees a more stable order pattern from the retailer. The predictability afforded by increased supply and higher order stability lowers the overall risk for the local farmer and allows him to better manage staffing (pick and pack lines) and transportation. In essence, through coordination and forecast visibility, an organization such as Red Tomato can pool the resources from multiple local farms, making them appear like a large, reliable and hence more attractive supplier to the retailer.

Backhauling. An operational lever that could reduce the logistical cost of sourcing from the local farm is backhauling. For example, Walmart's local food program (Heritage Agriculture) leverages its ability to backhaul local produce from farms that are located between its distribution centers (DC's) and stores. The retailer regularly sends loaded trucks from her DC's to her stores. On the way back from the stores, the trucks are empty. Using this truck capacity to transport produce from farms to the DC significantly reduces the logistics cost of sourcing from the local farm.

Let b be the reduced unit ordering cost under backhauling from the local farm. At time $t-1$, the retailer solves the following optimization problem to determine the optimal quantity to order from the mainstream farm, y_m:

$$\min_{y_m \geq 0} (w-b)\, y_m + \mathbb{E}_{\epsilon_1, \epsilon_s}\left[\min_{y_m+\mu+\epsilon_s \geq y \geq y_m} b\, \mathbb{E}_{\epsilon_2}[(y-\lambda-\epsilon_1-\epsilon_2)^+] + (r-b)\, \mathbb{E}_{\epsilon_2}[(\lambda+\epsilon_1+\epsilon_2-y)^+] \right]. \quad (6)$$

Equation (6) is analogous to Equation (4), except that in the inner minimization, the retailer's unit ordering cost is b instead of w. There is also an additional $(w-b)\, y_m$ term in the outer minimization which accounts for the additional cost of the units ordered from the mainstream farm.

The following proposition presents the retailer's profit-maximizing order quantity from the mainstream farm.

Proposition 5 *The retailer's profit-maximizing order quantity from the mainstream farm y_m^* solves* *the following*

$$\left(\frac{w-b}{w-r}\right)\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m-\sigma_2 z-\lambda)/\sigma_1}^{(y_m+\mu+v\sigma_s-\sigma_2 z-\lambda)/\sigma_1}\phi(u)du\phi(v)dv$$

$$\int_{-\infty}^{(y_m-\sigma_2 z-\lambda)/\sigma_1}\big[1-\frac{r}{r-w}\,\Phi\big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\big)\big]\,\phi(u)du$$

$$+\Phi\big(\frac{-\mu}{\sigma_s}\big)\int_{(y_m-\sigma_2 z-\lambda)/\sigma_1}^{\infty}\big[1-\frac{r}{r-w}\,\Phi\big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\big)\big]\,\phi(u)du$$

$$+\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z-\lambda)/\sigma_1}^{\infty}\big[1-\frac{r}{r-w}\,\Phi\big(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2}\big)\big]\,\phi(u)du\phi(v)dv=0, \quad (7)$$

where $z=\Phi^{-1}\left(\frac{r-b}{r}\right)$.

The retailer's profit-maximizing local order is similar to what is presented in Proposition 1. The only change is due to the reduction of the cost of sourcing from local farms from w to b.

Based on the mainstream order presented implicitly in Proposition 5, we can show the effect of backhauling on the local farmer.

Proposition 6 *As the unit ordering cost under backhauling b decreases,*

1. *the average amount the retailer orders from the local farm increases,*
2. *the profit-maximizing order quantity from the mainstream farm y_m^* decreases,*
3. *the average amount sold by the local farm increases,*
4. *the retailer's optimal expected one period cost decreases.*

The backhauling mechanism works differently than coordination to improve conditions for the local farmer. Instead of allowing the retailer to better manage mismatch cost, backhauling increases the per unit margin of the retailer. Without backhauling, a local tomato is the same as a mainstream tomato. With backhauling, the retailer's margin on local tomatoes is higher. Therefore, the retailer prefers to sell more local tomatoes. Thus, as shown in Proposition 6, she orders less from mainstream and more from local. However, our numerical analysis in Section 5 shows that the retailer stocks out more because there is local supply uncertainty. Therefore, with backhauling, the retailer's mismatch cost can actually increase, but her average margin on each unit sold increases, allowing her to increase profit.

Manager's Discretion. Unlike coordination and backhauling, which are operational levers, *manager's discretion* is an order policy that the retailer can choose to follow in order to support local farmers. A number of successful grocery chains have local sourcing policies, including Whole Foods, Walmart, Wegmans, and Thrifty Foods. We consider an extreme case where the retailer exercises *manager's discretion* and commits to buy everything the local supplier produces. This is an agreement that Walmart has with some farmers in its Heritage Agriculture Program. Although she buys everything the local farm produces, the retailer must still order from the mainstream supplier because she knows that there probably will not be enough local supply to satisfy demand. In this section, we derive the impact of that policy on the retailer's profit.

Taking into account the supply of local produce that will be available in period t, the retailer orders the following from the mainstream farm:

Proposition 7 *Using manager's discretion, the retailer maximizes profit by ordering the following quantity from the mainstream farm:*

$$y_m^* = \lambda - \mu + \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_S^2}\ \Phi^{-1}(\frac{r-w}{r}). \tag{8}$$

Inspection of Equation (8) shows that for the same service level outcome, i.e., $\frac{r-w}{r}$, the retailer carries more safety stock under manager's discretion than under the mainstream only order policy (Lemma 1, Equation (2)). This results in the following corollary.

Corollary 1 *Using manager's discretion, the retailer's profit decreases compared to the mainstream only or hybrid sourcing (without manager's discretion) order policies.*

By committing to buy whatever the local farm produces, the retailer loses the operational advantage of deciding her local order based on a clearer demand signal. Moreover, she must account for local supply uncertainty when she orders from the mainstream farm, thus increasing the amount of safety stock she carries. Clearly, manager's discretion is good for the local farmer (guaranteed sales), but the retailer is worse off than even if she orders from mainstream only.

4 Target Service Level

In the grocery retailing business, a more common objective is to maintain a certain service level rather than to maximize profit. In particular, for staple produce items such as tomatoes or bananas, it is important for the retailer to maintain high service levels because consumers have come to expect these items to be in-stock. Therefore, in this section, we consider the problem where the retailer optimizes her profit subject to a service level constraint. We focus on type 1 service level, the probability of being in stock. The desired service level is denoted by ρ.

Using the mainstream only sourcing policy, the retailer's optimal order can be obtained by

$$\hat{y} = \lambda + \sqrt{\sigma_1^2 + \sigma_2^2}\, \Phi^{-1}\left(\max\{\rho, \frac{r-w}{r}\}\right). \tag{9}$$

In the hybrid sourcing case, the retailer's optimal order policy follows from a constrained optimization problem. To state the retailer's problem, let $\mathbf{y} = (y_m, y_l)$ denote her order policy with the understanding that y_m and y_l are chosen sequentially. In particular, the local order quantity y_l is chosen after observing $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_s$. Also denote the retailer's cost under the order policy $\mathbf{y}$ by $C(\mathbf{y})$:

$$C(\mathbf{y}) = \mathbb{E}_{\epsilon_1,\epsilon_s,\epsilon_2}\Big[w(y_m+\min(y_l,(\mu+\epsilon_s)^+)-\lambda-\epsilon_1-\epsilon_2)^+ + (r-w)\,(\lambda+\epsilon_1+\epsilon_2-y_m-\min(y_l,(\mu+\epsilon_s)^+))^+\Big]. \tag{10}$$

Similarly, denote the service level under order policy $\mathbf{y}$ by $S(\mathbf{y})$:

$$S(\mathbf{y}) = \mathbb{P}\big(\lambda + \epsilon_1 + \epsilon_2 \le y_m + \min(y_l, (\mu+\epsilon_s)^+)\big). \tag{11}$$

Then the retailer's problem is to choose $\mathbf{y}$ (with $y_m, y_l \ge 0$) so as to

$$\text{Min } C(\mathbf{y}) \ \text{ subject to } \ S(\mathbf{y}) \ge \rho. \tag{12}$$

To solve this problem, we consider the following relaxed problem in which the constraint is replaced by a penalty term in the objective: For $q > 0$, choose $\mathbf{y}$ to

$$\text{Min } C(\mathbf{y}) - q\, S(\mathbf{y}). \tag{13}$$

Letting[2] $\mathbf{y}(q) = \text{argmin}\{C(\mathbf{y}) - q\, S(\mathbf{y})\}$, the following lemma shows that higher penalties lead to

[2]It is straightforward to argue that the objective is strictly convex, and hence, the optimal solution is unique, and that $\mathbf{y}(q)$ is well defined.

higher service levels.

Lemma 2 *As q increases, $S(\mathbf{y}(q))$ increases too.*

To facilitate the solution to the retailer's problem (12), define

$$\underline{\rho} = S(\mathbf{y}(0)) \;\text{ and }\; \overline{\rho} = \lim_{q\to\infty} S(\mathbf{y}(q)). \tag{14}$$

Then for $\rho \in (\underline{\rho}, \overline{\rho})$, let $q(\rho)$ be such that[3] $S(\mathbf{y}(q(\rho))) = \rho$. The following proposition provides a solution to the retailer's problem (12).

Proposition 8 *The order policy $\mathbf{y}(q(\rho))$ is an optimal solution for (12) and constitutes an optimal order policy for the retailer.*

Next, we characterize the optimal order quantity $\mathbf{y}(q(\rho))$ further. Note that the probability $S(\mathbf{y})$ of being in stock can be written as follows:

$$\begin{aligned} S(\mathbf{y}) &= \mathbb{E}_{\epsilon_1,\epsilon_s}\left[\mathbb{P}\big(\lambda + \epsilon_1 + \epsilon_2 \le y_m + \min(y_l, (\mu+\epsilon_s)^+)\big)\right] \\ &= \mathbb{E}_{\epsilon_1,\epsilon_s}\left[\Phi\big(\frac{\min(y_l,(\mu+\epsilon_s)^+) + y_m - \lambda - \epsilon_1}{\sigma_2}\big)\right]. \end{aligned} \tag{15}$$

The formulation (13) can be rewritten as follows:

$$\begin{aligned} \min_{y_m\ge 0} \mathbb{E}_{\epsilon_1,\epsilon_s}\Bigg[&\min_{\mu+\epsilon_s\ge y_l\ge 0} w\,\mathbb{E}_{\epsilon_2}[(y_m+y_l-\lambda-\epsilon_1-\epsilon_2)^+] + (r-w)\,\mathbb{E}_{\epsilon_2}[(\lambda+\epsilon_1+\epsilon_2-y_m-y_l)^+] \\ &- q\Phi\big(\frac{y_l+y_m-\lambda-\epsilon_1}{\sigma_2}\big)\Bigg]. \end{aligned} \tag{16}$$

Then we consider the inner optimization (or, the second period) problem given $y_m, \tilde{\epsilon}_s, \tilde{\epsilon}_1$:

$$\begin{aligned} \min_{\mu+\tilde{\epsilon}_s\ge y_l\ge 0} &w\,\mathbb{E}_{\epsilon_2}[(y_m+y_l-\lambda-\tilde{\epsilon}_1-\epsilon_2)^+] + (r-w)\,\mathbb{E}_{\epsilon_2}[(\lambda+\tilde{\epsilon}_1+\epsilon_2-y_m-y_l)^+] \\ &- q\Phi\big(\frac{y_l+y_m-\lambda-\tilde{\epsilon}_1}{\sigma_2}\big). \end{aligned} \tag{17}$$

Let $z(q)$ denote the unique solution of the following equation:

$$\frac{r-w}{r} = \Phi(z) - \frac{q}{\sigma_2 r}\phi(z). \tag{18}$$

The following proposition characterizes the optimal solution to (17).

[3]It is clear from Lemma 2 that $S(y(\cdot))$ is invertible and that $q(\rho)$ is well defined.

Proposition 9 *Let $y = \lambda + \tilde{\epsilon}_1 + \sigma_2 z(q(\rho))$. Then the retailer's profit maximizing order quantity from the local farm is $y_l = (y - y_m)^+$.*

Note that the unconstrained order quantity $y = \lambda + \tilde{\epsilon}_1 + \sigma_2 z(q(\rho))$ can be interpreted as the order quantity in a simple newsvendor setting with the optimal service level $\hat{\rho}$:

$$y = \lambda + \tilde{\epsilon}_1 + \sigma_2 \Phi^{-1}(\hat{\rho}), \tag{19}$$

where $\hat{\rho} = \Phi(z(q(\rho)))$. Therefore, when ordering from the local farmer the retailer strives to achieve a service level $\hat{\rho}$, which does not depend on the realizations of ϵ_1 and ϵ_s (though the quantity delivered from the local farm depends on both). This observation simplifies the numerical solution of the retailer's problem (12). Namely, it suffices to search over y_m and $\hat{\rho}$. For each such pair, one computes the cost $C(\mathbf{y})$ and the service level $S(\mathbf{y})$. Then restricting attention to those pairs $(y_m, \hat{\rho})$ such that $S(\mathbf{y}) = \rho$, the pair with the lowest cost gives the optimal order policy for the retailer.

Lastly, observe that whenever the service level constraint binds, i.e., $q > 0, \hat{\rho} > (r - w)/r$, which follows from (18). This merely means that the retailer seeks to achieve a service level higher than the optimal service level in the simple newsvendor model. Interestingly, we also observe that $\hat{\rho} > \rho$. Otherwise, if $\hat{\rho} \leq \rho$, because of the capacity constraint and the resulting truncation of local orders, we would have $S(\mathbf{y}) < \rho$, violating the service level constraint. In other words, recognizing the local capacity constraint, the retailer seeks a higher service level $\hat{\rho}$ in the second stage problem than the required service level ρ. This enables her to achieve the service level ρ on average.

In Section 5, we apply these findings in a numerical example to show how coordination and back-hauling affect retail profit and local farm operating conditions when the retailer's objective is to achieve a target service level. If the retailer implements hybrid sourcing with the manager's discretion order policy, following the same logic as the mainstream only optimal quantity in Equation (9), she orders the following quantity from the mainstream farm:

$$\hat{y}_m = \lambda - \mu + \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_s^2}\, \Phi^{-1}\left(\max\left\{\rho, \frac{r - w}{r}\right\}\right). \tag{20}$$

Comparing Equations (9) and (20), we see that as in Section 3 (profit-maximization), the retailer carries more safety stock when she implements manager's discretion, and as a result her profit using hybrid sourcing with manager's discretion is lower than her profit when she sources from mainstream only.

Combining manager's discretion and an operational mechanism. The manager's discretion order policy is clearly the most effective mechanism for improving local farm operating conditions, however, it comes at the expense of retail profit. The question is whether combining manager's discretion and an operational mechanism (coordination or backhauling) can guarantee local farm order stability and increase retail profit above the mainstream only policy?

The combination of manager's discretion with coordination will at best increase the retailer's profit to the same level as sourcing from mainstream only. This is because under manager's discretion, coordination can only reduce supply uncertainty - demand visibility is irrelevant because the retailer has already committed to buying everything from the local farm. Thus, coordination can at best make local supply deterministic, which makes local supply simply an extension of the mainstream supply. However, if backhauling were possible and it reduced the ordering cost of local produce enough, the retailer could more than compensate for the profit decrease resulting from committing to the manager's discretion order policy.

Proposition 10 *Suppose the retailer's objective is to achieve service level ρ and she implements manager's discretion. The retailer's profit increases as the unit ordering cost under backhauling b decreases. If backhauling reduces the unit ordering cost enough, the retailer's profit is higher than her profit under mainstream only and under hybrid sourcing.*

Implementing manager's discretion with low enough ordering cost under backhauling makes sourcing locally not only viable, but more profitable than sourcing from mainstream only. Backhauling and manager's discretion are complementary mechanisms: backhauling reduces the cost of local food and manager's discretion increases the percentage of local food purchased by the retailer. Thus, as ordering cost decreases, the retailer's average margin increases. This is the winning combination used by Walmart in its Heritage Agriculture program.

	profit maximization		target service level 0.99	
	mainstream only	hybrid sourcing	mainstream only	hybrid sourcing
mainstream order, y_m^*	1983	1898	2465	2336
avg local order	–	120	–	47
avg retail service level	0.46	0.47	0.99	0.99
avg retail profit ($)	1281.12	1307.60	1026.80	1100.05
avg retail mismatch cost ($)	118.88	92.40	373.20	299.95
avg retail overage cost ($)	57.16	43.85	372.74	299.46
avg retail underage cost ($)	61.73	48.54	0.46	0.49
avg local farm utilization	–	0.49	–	0.24
coeff. var. local farm util	–	0.85	–	1.53
avg local farm service level	–	0.72	–	0.90

Table 1: Retail performance and local farm operating conditions under mainstream only and hybrid sourcing policies when the retailer's objective is to maximize profit or target service level of 0.99.

5 Numerical Example

We now use the analytical results derived in Sections 3 and 4 in a numerical example to illustrate the magnitude of the local food paradox and the impact of coordination, backhauling, and manager's discretion. We consider the example of stocking tomatoes at a retail grocery store, using parameter values that are consistent with an average suburban retail store. The average weekly demand for tomatoes is $\lambda = 2000$ pounds, and the uncertainty around the demand is captured in $\epsilon_1 \sim N(0, 160)$ and $\epsilon_2 \sim N(0, 120)$. We assume that the local supply is approximately 10% of demand, 200 pounds per week, but there is uncertainty that is captured by $\epsilon_s \sim N(0, 100)$. The retail price of tomatoes is $1.50 per pound and the retailer's unit cost is $0.80 per pound. The numerical results presented below are obtained using our analytical derivations for the retailer's mainstream and local orders, and monte carlo simulations for the outcome variables.

We first compare the results for the mainstream only and hybrid sourcing policies under the two objective functions, profit maximization and target service level (for high service level of 0.99). Looking at the two left columns of results for profit maximization in Table 1, we see that by using hybrid sourcing, the retailer increases profit by reducing her mismatch costs (mainstream only: profit = $1281.12 and mismatch cost = $118.88, hybrid sourcing: profit = $1307.60 and mismatch cost = $92.40). She is able to match demand better (lower mismatch cost) because the reduced leadtime of the local supplier enables better demand visibility. However, notice that the average local farm utilization is only 0.49, moreover, the coefficient of variation (CV) of utilization is 0.85. This is the manifestation of the local food paradox: the retailer uses the local supplier to react to demand volatility and as the result, the local farm sees a volatile order pattern. The local farm's utilization is low because he only receives orders when there is a higher than expected demand.

The effects of the local food paradox are exacerbated if the retailer's objective is to achieve a high service level (see Table 1, right two columns for service level objective). Because the retailer's objective is to achieve a high service level, she relies much more on the higher capacity, more reliable mainstream farm (profit maximization: $y_m^* = 1898$, target service level: $y_m^* = 2336$). The resulting average local order under hybrid sourcing decreases to 47 units from 120 units under profit maximization, and the coefficient of variation increases to 1.53 from 0.86. To achieve a service level of 0.99, the retailer incurs high overage cost (approximately \$300 compared to \$44 under profit maximization) and almost no underage cost (less than \$1), therefore, the impact of hybrid sourcing is primarily to reduce the retailer's overage cost. Basically, the local farm is used for the few unlikely occasions when demand is extremely high.

We now examine how coordination, backhauling, and manager's discretion affect the retailer's profit and the local farm's operating conditions. We will focus on the case where the retailer's objective is to achieve a high service level of 0.99. This is to be consistent with what we have observed in practice: that retail grocers strive for high service level because consumers have come to expect it.

Coordination. Table 2 shows the impact of coordinating multiple local farms. As the number of farms increases, local supply capacity increases from 200 units (1 farm) to 2000 units (10 farms) and the coefficient of variation of local supply decreases from 0.5 (1 farm) to 0.16 (10 farms). The good news is that coordination makes the retailer and the local farm better off. Retail profit increases from \$1100.06 (1 farm) to \$1176.05 (10 farms), local farm utilization increases from 0.23 (1 farm) to 0.28 (10 farms), and the coefficient of variation of local farm utilization decreases from 1.53 (1 farm) to 0.35 (10 farms).

The bad news is that local farm utilization remains quite low, even when average local supply is equal to average demand. This is the persistence of the local food paradox. As local supply capacity

target service level 0.99 local supply capacity (μ, σ_S)	**1 farm** $(200, 100)$	**2 farms** $(400, 141)$	**3 farms** $(600, 173)$	**5 farms** $(1000, 224)$	**10 farms** $(2000, 316)$
mainstream order, y_m^*	2336	2228	2136	1986	1740
avg local order	47	100	164	296	539
avg retail service level	0.99	0.99	0.99	0.99	0.99
avg retail profit ($)	1100.06	1140.65	1161.22	1173.97	1176.05
avg retail mismatch cost ($)	299.94	259.35	238.78	226.03	223.95
avg retail overage cost ($)	299.49	258.93	238.41	225.72	223.66
avg retail underage cost ($)	0.45	0.42	0.37	0.31	0.28
avg local farm utilization	0.23	0.27	0.29	0.31	0.28
coeff. var. local farm util	1.53	1.15	0.89	0.60	0.35
avg local farm service level	0.90	0.94	0.97	0.99	1.00

Table 2: The impact of coordinating multiple local farms on retail performance and local farm operating conditions.

increases and local supply uncertainty decreases, the distinction between local and mainstream supplies boils down to leadtime. The retailer will take advantage of the short local leadtime by using local supply to react to demand volatility. For each additional local unit, the benefit to the retailer decreases because the demand uncertainty associated with each additional local unit decreases. However, the supply uncertainty increases for each additional unit. Therefore, there is a threshold local quantity where the retailer's benefit from sourcing local is outweighed by the cost. When the retailer's objective is to achieve a high service level, this threshold quantity is even lower because the retailer faces the addition constraint that the local supply uncertainty be low enough to achieve the target service level. Thus, coordination helps to improve the operating conditions for the local farm, but the impact is limited.

Backhauling. Implementing backhauling when the retailer's objective is to achieve service level 0.99 is more helpful to the local farmer than coordination. In Table 3, we see that with backhauling, utilization increases from 0.24 to 0.41. However, the retailer cannot rely on the local farmer as much as it would like because local supply uncertainty would prevent the retailer from achieving its target service level, thus local farm utilization remains low.

In contrast, when the retailer maximizes profit, backhauling increases local farm utilization to a very high level. In Table 3 (bottom half), as unit ordering cost under backhauling decreases, local

target service level 0.99					
backhaul cost	**no backhaul**	**b**= \$0.7	**b**= \$0.5	**b**= \$0.3	**b**= \$0.1
mainstream order, y_m^*	2336	2333	2325	2322	2317
avg local order	47	50	61	69	95
avg retail profit (\$)	1100.05	1102.17	1105.97	1114.15	1123.13
avg retail mismatch cost (\$)	299.95	301.38	306.83	309.33	320.02
avg local farm utilization	0.24	0.25	0.29	0.32	0.41
coeff. var. local farm util	1.53	1.48	1.31	1.23	1.00
profit maximization					
backhaul cost	**no backhaul**	**b**= \$0.7	**b**= \$0.5	**b**= \$0.3	**b**= \$0.1
mainstream order, y_m^*	1898	1875	1832	1802	1785
avg local order	120	153	226	301	395
avg retail service level	0.47	0.47	0.47	0.47	0.47
avg retail profit	1307.60	1317.18	1342.13	1372.99	1408.28
avg retail mismatch cost	92.40	93.71	100.93	111.32	122.48
avg local farm utilization	0.48	0.59	0.75	0.87	0.95
coeff. var. local farm util	0.86	0.70	0.46	0.31	0.17

Table 3: The impact of backhauling on retail performance and local farm operating conditions.

farm utilization increases considerably (from 0.48 to 0.95) and the coefficient of variation decreases considerably (from 0.86 to 0.17). The retailer also benefits from backhauling – her profit increases as unit ordering cost under backhauling b decreases, however, note that the retailer's mismatch cost *increases* as backhaul cost decreases. The backhaul mechanism works through the retailer's margin, which in turn affects her profit. Thus as the margin increases (i.e., the unit ordering cost b decreases), the retailer uses a higher fraction of local supply to fill demand. Increased margin means that the retailer carries more safety stock, therefore, overage cost increases. Local supply is also uncertain so underage cost increases. Thus, overall mismatch cost increases. Although profit maximization is not an appropriate objective for staple produce items, these insights can be applicable for specialty produce items (e.g., eggplant, pomegranate). Maintaining a high service level may not be necessary and thus the objective of the retailer's stocking policy could be more closely aligned with profit maximization.

Manager's discretion. By definition, manager's discretion increases the local farm's utilization to 1 and completely eliminates order volatility. This clearly benefits the local farmer, but retail profit decreases considerably. A retailer who follows the manager's discretion order policy while striving to achieve service level 0.99 will decrease profit to \$982.06. The retail profit under mainstream only and

target service level 0.99						
manager's discretion	**no**	**yes**	**yes**	**yes**	**yes**	**yes**
backhauling	**no**	**no**	**b**= \$0.7	**b**= \$0.5	**b**= \$0.3	**b**= \$0.1
mainstream order, y_m^*	2336	2320	2320	2320	2320	2320
avg local order	47	201	201	201	201	201
avg retail profit (\$)	1100.05	982.06	1002.15	1042.32	1082.49	1122.66
avg retail revenue (\$)	2999.04	2998.89	2998.89	2998.89	2998.89	2998.89
avg retail cost (\$)	1898.98	2016.83	1996.74	1956.57	1916.40	1876.24
avg retail mismatch cost (\$)	299.94	417.94	413.50	404.63	395.77	386.90

Table 4: The impact of backhauling and manager's discretion combination on retail performance.

hybrid sourcing without manager's discretion are \$1026.80 and \$1100.05, respectively. This decrease in profit results from an increase in mismatch cost to \$417.94 (vs. \$373.20 for mainstream only and \$299.94 for hybrid sourcing without manager's discretion).

Combining manager's discretion and backhauling. Table 4 shows that as the unit ordering cost under backhauling b decreases, the retailer can actually make more profit than sourcing from mainstream only. In fact, manager's discretion with backhauling can result in higher profit than even hybrid sourcing without manager's discretion and backhauling. In this scenario, the fact that backhauling only affects retail profit, not optimal order quantities, is advantageous.

6 Impact on the Environment: Food Miles and Food Waste

In this section, we explore two ways the proximity of local food to the consumer can affect environmental sustainability. An obvious area is the impact on food miles. Conceptualized by Tim Lang, food miles is the distance that food is transported from the time of production to consumption (Paxton 1994, Pirog and Benjamin 2005) – lower food miles is presumed to be better for the environment. Food miles has been criticized as being too simple to capture the complexities of environmental sustainability. For example, Weber and Matthews (2008) show that transportation contributes only 11% of life-cycle greenhouse gas emissions, with the majority coming from the production of food. Others argue for a more holistic assessment of the food supply chain that includes factors such as the livelihood of farmers in developing countries (Chi et al. 2009, Coley et al. 2009). Nevertheless, food miles remains a popular metric for environmental sustainability, primarily because of its simplicity and accessibility.

As mentioned in the introduction, we do not take a position on whether local food is better along any dimension. What our model does is quantify the increase in the percentage of produce sourced locally as a result of the hybrid sourcing policy, and coordination, backhauling, and manager's discretion mechanisms. The first-order effect on food miles is simply that hybrid sourcing lowers average food miles of a produce item by incorporating a local supplier. Moreover, coordination, backhauling, and manager's discretion all increase the percentage of local food, further reducing the average food miles. However, what our analysis also shows is that the *total* amount of produce ordered changes, thus changing total food miles. We can also show the expected sales to the consumer. Therefore, a metric such as the average food mile per unit of produce consumed could be calculated. Thus, our results can add more nuance to the food miles metric by incorporating supply chain dynamics.

Extended Shelf Life. Another potential environmental benefit of local food is the reduction of food waste at the retailer. Because local food spends less time in transport, if handled correctly (i.e., proper and timely refrigeration), the available shelf life of local fresh produce could be longer than mainstream fresh produce. We develop two order heuristics for the retailer and show that leveraging extended shelf life of local food can reduce retail waste, increase retail profit, and improve local farm operating conditions. We continue with the setting introduced in Section 2, however, we assume that the shelf life of local produce is two periods. Based on common retail grocery operating practices, we assume that the produce is sold on a first-in-first-out basis.

The combination of perishability, different leadtimes and shelf lives for local and mainstream fresh produce, and the local supplier capacity constraint make the exact analysis of the extended shelf life scenario intractable. Therefore, in what follows we will make several simplifications and approximations to motivate heuristic solutions for the retail order policy. First, defining $x(t)$ as the on-hand inventory at time t prior to ordering from the local farm, we assume for simplicity that $x(t) \leq D(t)$ for all t. This merely ensures that demand is high enough to avoid any wastage of produce. Second, we assume that the local farmer is uncapacitated. Under this assumption, without loss of optimality, the retailer orders only from the local farmer. Therefore, in essence, the problem reduces to a single echelon inventory problem with a single non-perishable product ordered from an uncapacitated supplier under lost sales. The lost sales penalty is $p > 0$ per unit, and the holding cost rate is $h > 0$.

Denote the retailer's demand forecast for period t when ordering from the mainstream farm by $D(t)$. Letting $\tilde{D}(t)$ denote the demand forecast (conditional expectation of $D(t)$) at time t, i.e., $\tilde{D}(t) = \lambda + \tilde{\epsilon}_1 + \epsilon_2$, the on-hand inventory at time $t+1$ before ordering from the local farm is given by $x(t+1) = [y(t) - \tilde{D}(t)]^+$, where $y(t)$ is the inventory in period t after the shipment from the local farm arrives but before the demand is realized.

An order policy of the retailer for the entire planning horizon is denoted by π. At the start of period t, the retailer chooses the amount to order from the local farm, $y(t) - x(t)$, to minimize total expected discounted costs under policy π at time t, given by

$$C_t(x|\pi) = \sum_{t=0}^{\infty} \alpha^t \, \mathbb{E}\big[w(y(t) - x(t)) + L(y(t))|x(t)\big], \tag{21}$$

where the expectation is over all possible demand sequences, α is the discount factor and $L(y) = p\mathbb{E}_{\epsilon_2}[\,(\tilde{D} - y)^+] + h\mathbb{E}_{\epsilon_2}[(y - \tilde{D})^+]$ is the mismatch cost. A straightforward manipulation of (21) (provided in Appendix C) yields

$$C_t(x|\pi) = -wx(t) + \sum_{t=0}^{\infty} \alpha^t \mathbb{E}[L^+(y(t))], \tag{22}$$

where $L^+(y) = L(y) + wy - \alpha w \mathbb{E}_{\epsilon_2}[(y - \tilde{D})^+]$. Then, it follows from the analysis of Section 3.3 of Heyman and Sobel (2004) that a *myopic policy* is optimal (see also pp. 374–387 of Zipkin (2000) for an alternative derivation). Moreover, the cost function $L^+(y)$ serves as the current period's cost. Then, defining $y^+ = \inf \operatorname{argmin}_y \mathbb{E}[L^+(y)]$, the optimal ordering policy from the local farm is a base-stock policy with the base-stock level of $\tilde{y} = y^+$ provided $y(t) \geq x(t)$. The base-stock level[4] is characterized in Section 9.4.6 of Zipkin (2000) as $\tilde{y} = \lambda + \tilde{\epsilon}_1 + \sigma_2 \Phi^{-1}\big(\frac{p-w}{p+h-\alpha w}\big)$.

Motivated by this characterization, we develop two heuristics, both of which use a base-stock policy when ordering from the local farm. The base-stock level is given by $\tilde{y} = \hat{y} + \tilde{\epsilon}_1$, where $\hat{y}$ is a constant term to be determined and $\tilde{\epsilon}_1$ is the forecast update prior to ordering from the local farm.

The two heuristics differ in the policy for orders from the mainstream farm. In the first heuristic, the retailer orders a constant amount y_m from the mainstream farm at every period regardless of the

[4]Zipkin (2000) also argues that a base-stock policy is also optimal (with a different base-stock level) when the supply is capacitated; see pp. 409–410.

target service level 0.99			
extended shelf life	**no**	**heuristic 1**	**heuristic 2**
avg mainstream order	2336	2285	2157
avg local order	47	139	263
avg retail profit ($)	1100.05	1113.82	1140.35
avg retail mismatch cost ($)	299.95	286.21	259.67
avg retail overage cost ($)	299.46	285.30	258.80
avg retail underage cost ($)	0.49	0.52	0.51
avg local farm utilization	0.24	0.34	0.65
profit maximization			
extended shelf life	**no**	**heuristic 1**	**heuristic 2**
avg mainstream order	1898	1798	1784
avg local order	120	277	808
avg retail service level	0.47	0.62	0.65
avg retail profit ($)	1307.60	1325.06	1328.02
avg retail mismatch cost ($)	92.40	74.96	72.00
avg retail overage cost ($)	43.85	34.06	33.57
avg retail underage cost ($)	48.54	40.86	38.39
avg local farm utilization	0.49	0.67	1.00

Table 5: The impact of extended shelf life order heuristics on retail performance and local farm operating conditions when the retailer's objective is to maximize profit or target service level of 0.99. Expected discounted profit and costs are calculated for the heuristics and then multiplied by (1 - discount factor) to derive average per period profit and cost.

state. Therefore, we search over $\hat{y}$ and y_m to select the $(\hat{y}, y_m)$ pair which results in the lowest total expected discounted cost of the original dynamic programming problem. In the second heuristic, the retailer uses her knowledge of $x(t)$, $y(t)$ and $\tilde{D}(t)$ in choosing $y_m(t)$, the amount ordered from the mainstream farm at time t. The retailer's objective is to choose $y_m(t)$ to bring $x(t+1)$, the inventory at time $t+1$ prior to ordering from the local farm, to a predetermined inventory level. That is, we need to specify a second target inventory level in this heuristic for mainstream orders, denote it by $\hat{y}_m$. Then, we search over $(\hat{y}, \hat{y}_m)$ pairs to find the pair with the lowest total expected discounted cost.

Numerical Example: Extended Shelf Life. Continuing with the parameters from the numerical example in Section 5, we illustrate the performance of the two extended shelf life heuristics under target service level and profit maximization objectives. In Table 5, we see that both extended shelf life heuristics perform better than hybrid sourcing without extended shelf life on retail profit, local farm utilization, and retail service level (under the profit maximization objective). Extended shelf life increases the probability of selling a given local unit, thus the expected margin on a local unit increases. Similar to the backhaul mechanism, the retailer takes advantage of this increased margin by ordering more from the local supplier and reducing the amount ordered from the mainstream supplier. Moreover, by simply changing the order policy (without any other operational changes), the retailer's cost of overage decreases, i.e., the risk of incurring overage cost on a unit of local produce decreases, thus reducing food waste.

Focusing on the profit maximization objective (bottom half of Table 5), we see that heuristic 2 out-performs heuristic 1 (i.e., retail profit and local farm utilization are higher, and retail overage is lower under heuristic 2). In the first heuristic, the mainstream order remains constant over time, but in heuristic 2, both the mainstream and local orders can change every period. Notice that in heuristic 2, the local order increases dramatically (to 808 units) so that the local farm is fully utilized (but obviously the local farm cannot always fill the retailer's order). Because the expected margin of a local unit is higher than a mainstream unit, and the mainstream order can change every period, the retailer orders everything it can from the local farm and uses the mainstream supplier to make up the difference. Essentially, the retailer starts to use the mainstream supplier to react to demand volatility! Ordering a higher percentage of (local) units that have higher probability of being sold increases retail profit, increases local farm utilization, and decreases food waste. This effect is dampened in the target service level case (top half of Table 5) because the service level constraint and local farm uncertainty forces the retailer to continue relying on the mainstream supplier.

7 Conclusion

In this paper, we studied how the geographic constraints of the fresh produce supply chain resulted in the local food paradox. The proximity of local suppliers to the end consumers allows the retailer to react better to demand volatility. However, the resulting local order volatility increases the risk and makes operating conditions very difficult for the local farm.

We studied three mechanisms that are used in practice to incorporate local food into the retailer's sourcing policy: coordination, backhauling, and manager's discretion. Coordination and backhauling benefit both the retailer and the local farm. Coordination increases the effective capacity and decreases the supply uncertainty of the local farm, allowing the retailer to respond better to demand volatility thus lowering mismatch cost. Backhauling increases the retailer's margin on local produce. Thus, both mechanisms induce the retailer to order more from the local farm. Manager's discretion is obviously the most effective mechanism for improving operating conditions for the local farm, but profit suffers when the retailer commits to buying everything the local farm produces.

The effectiveness of these mechanisms also depend on the retailer's objective function. If the retailer's objective is to maintain a high service level, i.e., consistently in stock, the retailer has to rely even more on the mainstream supplier who is large and reliable, and only order from the local supplier in the very unlikely even that demand is extremely high. In this case, the effectiveness of the coordination mechanism is reduced because high service level forces the mismatch cost to be high. Backhauling is very effective when the retailer's objective is to maximize profit because it increases retail margin, but like coordination, it effect is dampened when the retailer's objective is to achieve a high service level.

However, we show that combining manager's discretion and backhauling can be a very effective way to incorporate local food into the retailer's sourcing strategy when she targets a high service level. We show that manager's discretion and backhauling are complementary mechanisms. The combination eliminates the local farm's risk of not selling its produce and for sufficiently low backhaul cost, increases the retailer's profit.

Local sourcing can also affect the environmental footprint of the food supply chain. Our results for the optimal order policy can be used to calculate the change in food miles when hybrid sourcing is implemented and potentially enrich the food miles metric by incorporating supply chain dynamics. We also show that upgrading local food transportation and handling processes to the same standards as the mainstream supply can reduce food waste by increasing local food shelf life. Moreover, the ordering heuristic we develop suggests that efforts to leverage extended shelf life can benefit the retailer and local farm.

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A Proofs of the Results in Section 3

Derivations of Equations (3) and (4) For period t arrivals, the retailer orders at time $t-1$ from the mainstream farm and at time t from the local farm. For generality, consider the scenario in which ordering costs from the local farm and the mainstream farm are different, denoted respectively by w_l and w_m; and sales revenue is r regardless of the source of produce.

First, consider the problem of ordering from the local farm at time t having committed to an amount y_m from the mainstream farm. Assuming unmet demand is lost and ignoring the constraint on local capacity the retailer's problem at time t is (cf. Porteus (2002))

$$\max_{y \geq y_m} \Pi(y) = r\big\{\lambda + \tilde{\epsilon}_1 - \mathbb{E}_{\epsilon_2}[(\lambda + \tilde{\epsilon}_1 + \epsilon_2 - y)^+]\big\} - w_l(y - y_m), \tag{23}$$

where the first term represents average revenue if the inventory level at the start of the period is y and the second term is the cost of ordering $y - y_m$ units from the local farm.

Since $y - \lambda - \tilde{\epsilon}_1 = \mathbb{E}_{\epsilon_2}[(y - \lambda - \tilde{\epsilon}_1 - \epsilon_2)^+] - \mathbb{E}_{\epsilon_2}[(\lambda + \tilde{\epsilon}_1 + \epsilon_2 - y)^+]$, equation (23) can be written as

$$\max_{y \geq y_m} \Pi(y) = (r - w_l)(\lambda + \tilde{\epsilon}_1) - [L(y) - w_l y_m]\,, \tag{24}$$

where $L(y) = w_l \mathbb{E}_{\epsilon_2}[(y - \lambda - \tilde{\epsilon}_1 - \epsilon_2)^+] + (r - w_l)\mathbb{E}_{\epsilon_2}[(\lambda + \tilde{\epsilon}_1 + \epsilon_2 - y)^+]$.

An equivalent problem to that of Equation (24) is given by

$$\min_{y \geq y_m} L(y) - w_l y_m. \tag{25}$$

Ignoring the y_m term without loss of optimality reduces this formulation to a classical newsvendor problem with the constraint $y \geq y_m$. The optimal solution to the unconstrained problem is

$$\tilde{y} = \lambda + \tilde{\epsilon}_1 + \sigma_2 z^*, \tag{26}$$

where $z^* = \Phi^{-1}\left(\frac{r - w_l}{r}\right)$ is the critical fractile. Moreover, setting $w_l = w$ yields the statement in (3). The optimal solution is derived more formally in the proof of Proposition 1.

At time $t-1$, the retailer finds optimal order quantity y_m from mainstream farms for period t arrival. Incorporating the capacity limit of the local farm in (25), retailer's problem is given by

$$\min_{y_m \geq 0} (w_m - w_l)\, y_m + \mathbb{E}_{\epsilon_s}\Bigg[\min_{y_m + \mu + \epsilon_s \geq y \geq y_m} w_l\, \mathbb{E}_{\epsilon_1,\epsilon_2}[(y - \lambda - \epsilon_1 - \epsilon_2)^+] + (r - w_l)\, \mathbb{E}_{\epsilon_1,\epsilon_2}[(\lambda + \epsilon_1 + \epsilon_2 - y)^+]\Bigg]. \tag{27}$$

Setting $w_m = w_l = w$ yields Equation (4).

Alternative Representations of Equations (3) and (4) For the analysis in the appendix we use an alternative representation of the retailer's problem at time t rather than the one in (25). That is, the retailer solves $\min_{y \geq y_m} w_l\, y + r\, \mathbb{E}_{\epsilon_2}[(\lambda + \tilde{\epsilon}_1 + \epsilon_2 - y)^+]$. Note that, this follows directly from Equation (23). Similarly, the retailer's problem at time $t-1$ can be represented by

$$\min_{y_m \geq 0} (w_m - w_l)\, y_m + \mathbb{E}_{\epsilon_s}\Bigg[\min_{y_m + \mu + \epsilon_s \geq y \geq y_m} w_l\, y + r\, \mathbb{E}_{\epsilon_1,\epsilon_2}[(\lambda + \epsilon_1 + \epsilon_2 - y)^+]\Bigg]. \tag{28}$$

Proof of Lemma 1. Define $D = \lambda + \epsilon_1 + \epsilon_2$ as a normal random variable with mean λ and variance $\sqrt{\sigma_1^2 + \sigma_2^2}$. Then, the retailer's problem is given by $\min_{y \geq 0} w\, \mathbb{E}_D[(y - D)^+] + (r - w)\, \mathbb{E}_D[(D - y)^+]$, and the newsvendor solution is $y = \lambda + \sqrt{\sigma_1^2 + \sigma_2^2}\Phi^{-1}\left(\frac{r-w}{r}\right)$. ■

Proof of Proposition 1. Define $L(y) = w\, \mathbb{E}_{\epsilon_2}[(y - \lambda - \tilde{\epsilon}_1 - \epsilon_2)^+] + (r - w)\, \mathbb{E}_{\epsilon_2}[(\lambda + \tilde{\epsilon}_1 + \epsilon_2 - y)^+]$, which can be written as

$$L(y) = w \int_{-\infty}^{(y - \tilde{\epsilon}_1 - \lambda)/\sigma_2} (y - \tilde{\epsilon}_1 - \lambda - u\sigma_2)\phi(u)du + (r - w) \int_{(y - \tilde{\epsilon}_1 - \lambda)/\sigma_2}^{\infty} (\tilde{\epsilon}_1 + \lambda + u\sigma_2 - y)\phi(u)du.$$

Defining $H(x) = \int_0^x u\phi(u)du$, this can be written as

$$\begin{aligned}L(y) =& w\,[(y-\tilde{\epsilon}_1-\lambda)\Phi\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big) - \sigma_2 H\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big)]\\ &+ (r-w)\,[-\sigma_2 H\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big) - (y-\tilde{\epsilon}_1-\lambda) + (y-\tilde{\epsilon}_1-\lambda)\Phi\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big)]\\ =& r[(y-\tilde{\epsilon}_1-\lambda)\Phi\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big) - \sigma_2 H\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big)] - (r-w)(y-\tilde{\epsilon}_1-\lambda)\end{aligned}$$

The derivative of $L(y)$ with respect to y is

$$\frac{dL(y)}{dy} = r\,\Phi\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big) - r + w. \tag{29}$$

It follows that $d^2L(y)/dy^2 > 0$ and $L(y)$ is convex. Then, from (29) we write FOC as

$$r\,\Phi\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big) - r + w = 0.$$

Solving for $\Phi\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big)$ gives

$$\Phi\big(\frac{y-\tilde{\epsilon}_1-\lambda}{\sigma_2}\big) = \frac{r-w}{r},$$

which leads to optimal $\tilde{y}$

$$\tilde{y} = \tilde{\epsilon}_1 + \lambda + \sigma_2\Phi^{-1}\left(\frac{r-w}{r}\right).$$

Amount ordered from the local farm is simply the difference between the optimal inventory level at the start of the period and the amount already ordered from the mainstream farm. If the amount already ordered exceeds the optimal inventory level, then the retailer does not order from the local farm. ■

A.1 Proofs of Proposition 2, 3 and 5.

Auxiliary derivations. Recall that ϵ_1, ϵ_2 and ϵ_s are mean-zero normally distributed random variables with standard deviations of σ_1, σ_2 and σ_s, respectively. Define U, V and Z as standard normal random variables such that $\epsilon_1 = U\sigma_1$, $\epsilon_s = V\sigma_s$ and $\epsilon_2 = Z\sigma_2$. Define $v(y_m)$ as the objective of the retailer's problem at time $t-1$ given in (28) as

$$v(y_m) = (w_m - w_l)\,y_m + \mathbb{E}_{\epsilon_s}\left[\min_{y_m+\mu+\epsilon_s \geq y \geq y_m} w_l\,y + r\,\mathbb{E}_{\epsilon_1,\epsilon_2}[(\lambda+\epsilon_1+\epsilon_2-y)^+]\right]. \tag{30}$$

Through the change of variables introduced above this can be written as $v(y_m) = (w_m - w_l)\,y_m + \mathbb{E}_V\left[\min_{y_m+\mu+V\sigma_s\geq y\geq y_m} w_l\,y + r\,\mathbb{E}_{U,Z}[(\lambda+U\sigma_1+Z\sigma_2-y)^+]\right]$. Optimal y of the inner optimization problem is $\max[y_m, \min(y_m+\mu+V\sigma_s, \tilde{y})]$ where $\tilde{y}$ is given by (26) and can be written as $\tilde{y} = \lambda + u\sigma_1 + z^*\sigma_2$ in the new notation. Next, we write $v(y_m)$ for a range of u and v.

i. For $u < (y_m - \sigma_2 z^* - \lambda)/\sigma_1$, i.e. $\tilde{y} < y_m$,

$$v(y_m) = w_m y_m + r\int_{-\infty}^{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}\int_{(y_m-u\sigma_1-\lambda)/\sigma_2}^{\infty}(u\sigma_1+\lambda+z\sigma_2-y_m)\phi(z)dz\phi(u)du$$

ii. For $\tilde{y} > y_m$ and $\mu + v\sigma_s < 0$,

$$v(y_m) = w_m y_m + r\int_{-\infty}^{-\mu/\sigma_s}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}\int_{(y_m-u\sigma_1-\lambda)/\sigma_2}^{\infty}(u\sigma_1+\lambda+z\sigma_2-y_m)\phi(z)dz\phi(u)du\phi(v)dv$$

iii. For $\tilde{y} \in [y_m, y_m+\mu+v\sigma_s]$ and $\mu + v\sigma_s > 0$,

$$\begin{aligned} v(y_m) = w_m y_m + w_l &\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{(y_m+\mu+v\sigma_s-z^*\sigma_2-\lambda)/\sigma_1}(\lambda+u\sigma_1+z^*\sigma_2-y_m)\phi(u)du\phi(v)dv \\ &+ r\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{(y_m+\mu+v\sigma_s-z^*\sigma_2-\lambda)/\sigma_1}\int_{z^*}^{\infty}(z-z^*)\sigma^2\phi(z)dz\phi(u)du\phi(v)dv \end{aligned}$$

iv. For $\tilde{y} > y_m+\mu+v\sigma_s$ and $\mu + v\sigma_s > 0$,

$$\begin{aligned} v(y_m) &= w_m y_m + w_l\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}(\mu+v\sigma_s)\phi(u)du\phi(v)dv \\ &+r\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_1 u-\lambda)/\sigma_2}^{\infty}(\lambda+u\sigma_1+z^*\sigma_2-y_m-\mu-v\sigma_s)\phi(z)dz\phi(u)du\phi(v)dv \end{aligned}$$

Then, combining the statements for $v(y_m)$, which covers four parameter regimes, and collecting similar cost terms together we can write (30) as

$$\begin{aligned} v(y_m) &= w_m\, y_m \\ +w_l&\Bigg\{\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{(y_m+\mu+v\sigma_s-z^*\sigma_2-\lambda)/\sigma_1}(\lambda+u\sigma_1+z^*\sigma_2-y_m)\phi(u)du\phi(v)dv \\ &+\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}(\mu+v\sigma_s)\phi(u)du\phi(v)dv \\ +r&\Bigg\{\int_{-\infty}^{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}\int_{(y_m-u\sigma_1-\lambda)/\sigma_2}^{\infty}(u\sigma_1+\lambda+z\sigma_2-y_m)\phi(z)dz\phi(u)du \\ &+\int_{-\infty}^{-\mu/\sigma_s}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}\int_{(y_m-u\sigma_1-\lambda)/\sigma_2}^{\infty}(u\sigma_1+\lambda+z\sigma_2-y_m)\phi(z)dz\phi(u)du\phi(v)dv \\ &+\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{(y_m+\mu+v\sigma_s-z^*\sigma_2-\lambda)/\sigma_1}\int_{z^*}^{\infty}(z-z^*)\sigma^2\phi(z)dz\phi(u)du\phi(v)dv \\ &+\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_1 u-\lambda)/\sigma_2}^{\infty}(\lambda+u\sigma_1+z^*\sigma_2-y_m-\mu-v\sigma_s)\phi(z)dz\phi(u)du\phi(v)dv\Bigg\} \end{aligned}$$

Taking the derivative of this with respect to y_m and after several tedious but straightforward steps, which are omitted for brevity, we arrive at the following

$$\begin{aligned} \frac{dv(y_m)}{dy_m} &= (w_m - w_l)\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}\phi(u)du\phi(v)dv \\ &+\int_{-\infty}^{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}[(w_m-r)+r\,\Phi\big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\big)]\phi(u)du \\ &+\int_{-\infty}^{-\mu/\sigma_s}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}[(w_m-r)+r\,\Phi\big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\big)]\phi(u)du\phi(v)dv \\ &+\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}[(w_m-r)+r\,\Phi\big(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2}\big)]\phi(u)du\phi(v)dv. \end{aligned} \tag{31}$$

Denoting optimal order quantity from the mainstream farm by y_m^*

$$\frac{dv(y_m)}{dy_m}\bigg|_{y_m^*} = 0 \tag{32}$$

yields FOC. Dividing both sides of the equation in (32) by $w_m - r$ yields

$$\begin{aligned}\frac{dv(y_m)}{dy_m} &= \frac{w_m - w_l}{w_m - r}\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}\phi(u)du\phi(v)dv \\ &+ \int_{-\infty}^{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}[1+\frac{r}{w_m-r}\,\Phi(\frac{y_m-\lambda-u\sigma_1}{\sigma_2})]\phi(u)du \\ &+ \int_{-\infty}^{-\mu/\sigma_s}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}[1+\frac{r}{w_m-r}\,\Phi(\frac{y_m-\lambda-u\sigma_1}{\sigma_2})]\phi(u)du\phi(v)dv \\ &+ \int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}[1+\frac{r}{w_m-r}\,\Phi(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2})]\phi(u)du\phi(v)dv = 0.\end{aligned} \tag{33}$$

Next, consider $\partial^2 v(y_m)/\partial y_m^2$ when $w_m = w_l$. Setting $w_m = w_l = w$, equation (31) reduces to

$$\begin{aligned}\frac{\partial v(y_m)}{\partial y_m} &= \int_{-\infty}^{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}[(w-r)+r\,\Phi(\frac{y_m-\lambda-u\sigma_1}{\sigma_2})]\phi(u)du \\ &+ \Phi(\frac{-\mu}{\sigma_s})\frac{\partial}{\partial y_m}\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}[(w-r)+r\,\Phi(\frac{y_m-\lambda-u\sigma_1}{\sigma_2})]\phi(u)du \\ &+ \int_{-\mu/\sigma_s}^{\infty}\frac{\partial}{\partial y_m}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}[(w-r)+r\,\Phi(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2})]\phi(u)du\phi(v)dv.\end{aligned} \tag{34}$$

Using Leibniz's Rule $\partial^2 v(y_m)/\partial y_m^2$ can be written as

$$\begin{aligned}\frac{\partial^2 v(y_m)}{\partial y_m^2} &= \frac{1}{\sigma_1}\big[(w-r)+r\,\Phi(z^*)\big]\phi(\frac{y_m-\sigma_2 z-\lambda}{\sigma_1}) \\ &+ \int_{-\infty}^{(y_m-\sigma_2 z-\lambda)/\sigma_1}(\frac{r}{\sigma_2}\phi(\frac{y_m-\lambda-u\sigma_1}{\sigma_2})\big]\,\phi(u)du \\ &+ \Phi(\frac{-\mu}{\sigma_s})\frac{-1}{\sigma_1}\big[(w-r)+r\,\Phi(z^*)\big]\phi(\frac{y_m-\sigma_2 z-\lambda}{\sigma_1}) \\ &+ \Phi(\frac{-\mu}{\sigma_s})\int_{(y_m-\sigma_2 z-\lambda)/\sigma_1}^{\infty}\frac{r}{\sigma_2}\,\frac{1}{\sigma_2}\phi(\frac{y_m-\lambda-u\sigma_1}{\sigma_2})\big]\,\phi(u)du \\ &+ \int_{-\mu/\sigma_s}^{\infty}\frac{-1}{\sigma_1}\big[(w-r)+r\,\Phi(z^*)\big]\phi(\frac{y_m+\mu+v\sigma_s-\sigma_2 z-\lambda}{\sigma_1})\phi(v)dv \\ &+ \int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z-\lambda)/\sigma_1}^{\infty}\frac{r}{\sigma_2}\phi(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2})\big]\,\phi(u)du\phi(v)dv.\end{aligned} \tag{35}$$

The first, third and fifth terms on the right-hand side of (35) are zero because $\Phi(z) = (r-w)/r$. In addition, other terms are all positive. Thus, for $w_m = w_l$, $\partial^2 v(y_m)/\partial y_m^2 > 0$ and $v(y_m)$ is convex.

Proof of Proposition 2. Setting $w_m = w_l = w$ in (33) yields (5). ■

Proof of Proposition 3.

Part 1. The average amount ordered from the local farm increases as the optimal order quantity from the mainstream farm y_m^* decreases. Therefore, it suffices to show that y_m decreases as μ increases.

We want to show that $dy_m^*/d\mu < 0$. Using the implicit function theorem, we can write

$$\frac{\partial y_m^*}{\partial \mu} = -\frac{\frac{\partial^2 v(y_m)}{\partial y_m \partial \mu}}{\frac{\partial^2 v(y_m)}{\partial y_m^2}},$$

where $v(y_m)$ is the objective function of the retailer's problem at time $t-1$ given by Equation (4). From (35), we know that $\partial^2 v(y_m)/\partial y_m^2 > 0$ when $w_m = w_l$. It suffices to show that $\partial^2 v(y_m)/\partial y_m \partial \mu > 0$.

The statement for $\partial v(y_m)/\partial y_m$ for $w_m = w_l = w$ is given by equation (34). Taking the derivative of $\partial v(y_m)/\partial y_m$ with respect to μ yields

$$\begin{aligned}\frac{\partial^2 v(y_m)}{\partial y_m \partial \mu} &= -\frac{1}{\sigma_s}\phi\Big(\frac{-\mu}{\sigma_s}\Big)\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty} \Big[(w-r) + r\,\Phi\Big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\Big)\Big]\,\phi(u)du \\ &+\frac{\partial}{\partial \mu}\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}\Big[(w-r)+r\,\Phi\Big(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2}\Big)\Big]\,\phi(u)du\phi(v)dv. \qquad (36)\end{aligned}$$

Note that the first term in (34) does not depend on μ and thus its derivative is zero. Using the Leibniz's Rule on the second term of (36) allows us to rewrite (36) as

$$\begin{aligned}\frac{\partial^2 v(y_m)}{\partial y_m \partial \mu} &= -\frac{1}{\sigma_s}\phi\Big(\frac{-\mu}{\sigma_s}\Big)\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty} \Big[(w-r) + r\,\Phi\Big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\Big)\Big]\,\phi(u)du \\ &+\frac{1}{\sigma_s}\phi\Big(\frac{-\mu}{\sigma_s}\Big)\int_{(y_m-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty} \Big[(w-r) + r\,\Phi\Big(\frac{y_m-\lambda-u\sigma_1}{\sigma_2}\Big)\Big]\,\phi(u)du \\ &+\int_{-\mu/\sigma_s}^{\infty}\frac{\partial}{\partial \mu}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}\Big[(w-r)+r\,\Phi\Big(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2}\Big)\Big]\,\phi(u)du\phi(v)dv. \qquad (37)\end{aligned}$$

The first two terms in (37) cancel each other and using Leibniz's Rule on the third term gives

$$\begin{aligned}\frac{\partial^2 v(y_m)}{\partial y_m \partial \mu} &= \int_{-\mu/\sigma_s}^{\infty} -\big[(w-r)+r\,\Phi(z^*)\big]\phi\Big(\frac{y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda}{\sigma_1}\Big)\phi(v)dv \\ &-\int_{-\mu/\sigma_s}^{\infty}\int_{(y_m+\mu+v\sigma_s-\sigma_2 z^*-\lambda)/\sigma_1}^{\infty}\frac{r}{\sigma_2}\phi\Big(\frac{y_m+\mu+v\sigma_s-\lambda-u\sigma_1}{\sigma_2}\Big)\Big]\,\phi(u)du\phi(v)dv. \qquad (38)\end{aligned}$$

The first term on the right-hand side of (38) is zero since $\Phi(z^*) = (r-w)/r$ and the second one is positive. Therefore, $\partial^2 v(y_m)/\partial y_m \partial \mu > 0$, which completes the proof of part (i).

Part 2. Average amount sold by the local farm, call q, is the minimum of the retailer's order y_l^* and the available supply. That is, $q = \min(y_l^*, (\mu + \tilde{\epsilon}_s)^+)$. Clearly, q increases in μ since both terms in the minimum operator increases in μ. The fact that y_l^* increases in μ is shown in part (i) of this proposition.

Part 3. Increasing the mean local supply to $\tilde{\mu}$, while keeping mainstream orders at y_m^*, can reduce the retailer's cost as this is a relaxation on local supply constraint. At the minimum, the retailer can preserve her original optimal ordering policy, hence realizing the same expected one period cost as before. ■

Proof of Proposition 5. Setting $w_m = w$ and $w_l = b$ in Equation (33) yields (5). ■

A.2 Proofs of Proposition 4 and 6.

Auxiliary derivations. Again, for generality, we consider the scenario in which ordering costs from local farms and mainstream farms are different, denoted respectively by w_l and w_m. Define

$z^* = \Phi^{-1}\left(\frac{r-w_l}{r}\right)$. Then from Proposition 1, the optimal order quantity is $y_l^* = (\lambda + \tilde{\epsilon}_1 + \sigma_2 z^* - y_m)^+$.

Assume that i) $\lambda + \tilde{\epsilon}_1 + \sigma_2 z^* - y_m \geq 0$ almost surely and ii) $\mu + \tilde{\epsilon}_s \geq 0$ almost surely. Then the amount received from the local farm can be written as $\min(\lambda + \tilde{\epsilon}_1 + \sigma_2 z^* - y_m, \mu + \tilde{\epsilon}_s)$. The objective function in (25) (ignoring the y_m term without loss of optimality) is given by

$$L(z) = \sigma_2[w_l z + rI(z)], \tag{39}$$

where $z = \min(z^*, (y_m - \lambda + \mu + \tilde{\epsilon}_s - \tilde{\epsilon}_1)/\sigma_2)$ and $I(z) = \phi(z) - z(1 - \Phi(z))$. It is straightforward to show that $I(\cdot)$ is convex decreasing.

Then the objective function in (27) can be written as

$$v(y_m) = (w_m - w_l)y_m + \mathbb{E}_{\epsilon_1,\epsilon_s}[L(\min(z^*, (y_m - \lambda + \mu + \epsilon_s - \epsilon_1)/\sigma_2))]. \tag{40}$$

Define another random variable $X = \epsilon_s - \epsilon_1$ which is a mean-zero normally distributed random variable with variance $\sigma_x^2 = \sigma_1^2 + \sigma_s^2$. Then, defining $Z = X/\sigma_x$ as a standard normal variable (40) becomes

$$v(y_m) = (w_m - w_l)y_m + \mathbb{E}_Z\big[L\big(\min\big(z^*, \frac{y_m - \lambda + \mu + Z\sigma_x}{\sigma_2}\big)\big)\big], \tag{41}$$

and the derivative of $v(y_m)$ with respect to y_m is

$$\begin{aligned}\frac{\partial v(y_m)}{\partial y_m} &= (w_m - w_l) + \mathbb{E}_Z\left[L'\big(\frac{y_m - \lambda + \mu + Z\sigma_x}{\sigma_2}\big)\frac{1}{\sigma_2}; \frac{y_m - \lambda + \mu + Z\sigma_x}{\sigma_2} \leq z^*\right], \\ &= (w_m - w_l) + \int_{-\infty}^{(\lambda - y_m - \mu + z^*\sigma_2)/\sigma_x} L'\big(\frac{y_m - \lambda + \mu + z\sigma_x}{\sigma_2}\big)\frac{1}{\sigma_2}\phi(z)dz. \end{aligned} \tag{42}$$

From (39),

$$\begin{aligned} L'(z) &= \sigma_2[w_l + rI'(z)] \\ &= \sigma_2[w_l + r(-z\phi(z) - (1 - \Phi(z)) + z\phi(z)] \\ &= \sigma_2[w_l + r(-1 + \Phi(z))] \end{aligned} \tag{43}$$

From (42) and (43), the second derivative of $v(y_m)$ with respect to y_m can be derived as follows:

$$\frac{\partial^2 v(y_m)}{\partial y_m^2} = \frac{\partial}{\partial y_m}\int_{-\infty}^{(\lambda - y_m - \mu + z^*\sigma_2)/\sigma_x} [w_l + r(-1 + \Phi\big(\frac{y_m - \lambda + \mu + z\sigma_x}{\sigma_2}\big))]\phi(z)dz.$$

Then, using Leibniz Rule gives

$$\frac{\partial^2 v(y_m)}{\partial y_m^2} = -(w_l - r + r\Phi(z^*)))\phi\big(\frac{\lambda - y_m - \mu + z^*\sigma_2}{\sigma_x}\big) + \int_{-\infty}^{(\lambda - y_m - \mu + z^*\sigma_2)/\sigma_x} \frac{r}{\sigma_2}\phi\big(\frac{y_m - \lambda + \mu + z\sigma_x}{\sigma_2}\big)\phi(z)dz.$$

The first term is zero by the definition of z^* and the second term is always positive. Hence,

$$\frac{\partial^2 v(y_m)}{\partial y_m^2} > 0. \tag{44}$$

Proof of Proposition 4.

Part 1. The average amount ordered from the local farm increases as the optimal order quantity from the mainstream farm y_m decreases. Therefore, it suffices to show that y_m increases as σ_s increases. We want to show that $dy_m/d\sigma_s > 0$. Using the implicit function theorem, we can write

$$\frac{\partial y_m}{\partial \sigma_s} = -\frac{\frac{\partial^2 v(y_m)}{\partial y_m \partial \sigma_s}}{\frac{\partial^2 v(y_m)}{\partial y_m^2}},$$

where $v(y_m)$ is the objective function of the retailer's problem at time $t-1$ given by Equation (4). From (44) $\partial^2 v(y_m)/\partial y_m^2$ is positive. Define the random variable $X = \epsilon_s - \epsilon_1$ as before. In the remainder of this proof we will show that $\partial^2 v(y_m)/\partial y_m \partial \sigma_X$ is negative, which implies that $\partial^2 v(y_m)/\partial y_m \partial \sigma_s$ is negative.

From (42) we can write

$$\frac{\partial^2 v(y_m)}{\partial y_m \partial \sigma_X} = \frac{\partial}{\partial \sigma_X} \int_{-\infty}^{(\lambda - y_m - \mu + z^* \sigma_2)/\sigma_X} L'\big(\frac{y_m + \mu - \lambda + z\sigma_X}{\sigma_2}\big) \frac{1}{\sigma_2} \phi(z) dz$$

Using Leibniz Rule and the fact that $L'(z) = \sigma_2[w + r(-1 + \Phi(z))]$ yields

$$\begin{aligned}
\frac{\partial^2 v(y_m)}{\partial y_m \partial \sigma_X} &= \frac{\partial\big[\frac{\lambda - y_m - \mu + z^* \sigma_2}{\sigma_X}\big]}{\partial \sigma_X}(w - r + r\Phi(z^*))) + \int_{-\infty}^{(\lambda - y_m - \mu + z^* \sigma_2)/\sigma_X} \frac{\partial L'\big(\frac{y_m + \mu - \lambda + z\sigma_X}{\sigma_2}\big)/\sigma_2}{\partial \sigma_X} \phi(z) dz \\
&= \int_{-\infty}^{(\lambda - y_m - \mu + z^* \sigma_2)/\sigma_X} \frac{\partial\big[w - r + r\Phi\big(\frac{y_m + \mu - \lambda + z\sigma_X}{\sigma_2}\big)\big]}{\partial \sigma_X} \phi(z) dz \\
&= \int_{-\infty}^{(\lambda - y_m - \mu + z^* \sigma_2)/\sigma_X} \frac{zr}{\sigma_2}\ \phi\big(\frac{y_m + \mu - \lambda + z\sigma_X}{\sigma_2}\big) \phi(z) dz
\end{aligned}$$

Note that the expected order quantity from the local farm is $\lambda - y_m + z^* \sigma_2$. It is reasonable to assume that $\lambda - y_m + z^* \sigma_2 < \mu$. Thus $\frac{\partial^2 v(y_m)}{\partial y_m \partial \sigma_X}$ is negative.

Part 2. The local farm's sales is given by $q = \min(y_l^*, s)$, where $y_l^* = \lambda + z^* \sigma_2 - y_m^* + \epsilon_1$ and $s = \mu + \epsilon_s$ are normal random variables, and $z^* = \Phi^{-1}\left(\frac{r-w}{r}\right)$. Clark (1961) derives a formula for the maximum of two random variables. Defining $\tilde{\lambda} = \lambda + z^* \sigma_2 - y_m^*$, $\sigma = \sqrt{\sigma_1^2 + \sigma_s^2}$ and $\alpha = (\mu - \tilde{\lambda})/\sigma$, Equation (3) of Clark (1961) yields

$$\mathbb{E}(q) = \mu - \sigma[\alpha \Phi(\alpha) + \phi(\alpha)].$$

Note that it suffices to show that $\partial \mathbb{E}(q)/\partial \sigma_s < 0$. Letting $\alpha' = \partial \alpha/\partial \sigma_s$ and $\sigma' = \partial \sigma/\partial \sigma_s$, we have

$$\alpha' \sigma = \frac{\partial y_m^*}{\partial \sigma_s} - \alpha\, \sigma'. \tag{45}$$

Taking the derivative of $\mathbb{E}(q)$ with respect to σ_s gives

$$\frac{\partial \mathbb{E}(q)}{\partial \sigma_s} = -\sigma'[\alpha \Phi(\alpha) + \phi(\alpha)] - \sigma[\Phi(\alpha)\alpha' + \alpha\phi(\alpha)\alpha' - \alpha\phi(\alpha)\alpha']. \tag{46}$$

Rearranging and substituting (45 gives

$$\frac{\partial \mathbb{E}(q)}{\partial \sigma_s} = -\sigma' \phi(\alpha) - \Phi(\alpha) \frac{\partial y_m^*}{\partial \sigma_s},$$

which is negative since $\partial y_m^*/\partial \sigma_s > 0$.

Part 3. It suffices to show that $\partial v(y_m)/\partial \sigma_X > 0$. Setting $w_m = w_l = w$ in (41) yields

$$v(y_m) = \mathbb{E}_Z\big[L\big(\min\big(z^*, \frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2}\big)\big)\big],$$

and the derivative of $v(y_m)$ with respect to y_m is

$$\begin{aligned}
\frac{\partial v(y_m)}{\partial \sigma_X} &= \mathbb{E}_Z\left[L'\big(\frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2}\big)\frac{Z}{\sigma_2}; \frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2} \le z^*\right], \\
&= \mathbb{E}_Z\left[L'\big(\frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2}\big)\frac{Z}{\sigma_2}; Z \le \frac{\lambda - y_m - \mu + z^* \sigma_2}{\sigma_X}\right].
\end{aligned}$$

Note that

$$L'(z) = \sigma_2[w - r + r\Phi(z)] \le \sigma_2[w - r + r\Phi(z^*)] = \sigma_2[w - r + (r - w)] = 0,$$

and $Z < 0$ by the assumption also used in part (i): $\lambda - y_m + z^* \sigma_2 < \mu$. Hence $\partial v(y_m)/\partial \sigma_X > 0$

because the product of two negatives is positive. ∎

Proof of Proposition 6.

Part 1. Clearly, as b decreases the order-up-to-level $\tilde{y} = \lambda + \tilde{\epsilon}_1 + \sigma_2 \Phi^{-1}\left(\frac{r-b}{r}\right)$ increases. Also, as will be shown in *part 2*, the optimal order quantity from the mainstream farm y_m decreases as b decreases. Hence, the optimal order quantity from the local farm increases as b decreases.

Part 2. It suffices to show that $\partial y_m / \partial b > 0$. Using the implicit function theorem, we can write

$$\frac{\partial y_m}{\partial b} = -\frac{\frac{\partial^2 v(y_m)}{\partial y_m \partial b}}{\frac{\partial^2 v(y_m)}{\partial y_m^2}}.$$

We showed in (44) that $\partial^2 v(y_m)/\partial y_m^2$ is positive. In the remainder of this proof we will show that $\partial^2 v(y_m)/\partial y_m \partial b$ is negative. Setting $w_m = w$ and $w_l = b$ in the objective function as stated in (41) yields

$$v(y_m) = (w-b)y_m + \mathbb{E}_Z\big[L\big(\min\big(z^*, \frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2}\big)\big)\big],$$

and the derivative of $v(y_m)$ with respect to y_m is

$$\begin{aligned}
\frac{\partial v(y_m)}{\partial y_m} &= w - b + \mathbb{E}_Z\left[L'\big(\frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2}\big)\frac{1}{\sigma_2}; \frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2} \le z^*\right], \\
&= w - b + \mathbb{E}_Z\left[L'\big(\frac{y_m - \lambda + \mu + Z\sigma_X}{\sigma_2}\big)\frac{1}{\sigma_2}; Z \le \frac{\lambda - y_m - \mu + z^*\sigma_2}{\sigma_X}\right] \\
&= w - b + \frac{1}{\sigma_2}\int_{-\infty}^{(\lambda - y_m - \mu + z^*\sigma_2)/\sigma_X} L'\big(\frac{y_m - \lambda + \mu + z\sigma_X}{\sigma_2}\big)\phi(z)dz. \qquad (47)
\end{aligned}$$

Consider the following change of variable $\psi = \frac{y_m - \lambda + \mu + z\sigma_X}{\sigma_2}$. Then rewrite (47) as

$$\frac{\partial v(y_m)}{\partial y_m} = w - b + \frac{1}{\sigma_2}\int_{-\infty}^{z^*} L'(\psi) f_\psi(\psi) d\psi.$$

Then

$$\frac{\partial^2 v(y_m)}{\partial y_m \partial b} = -1 + \frac{1}{\sigma_2}\int_{-\infty}^{z^*} \frac{\partial}{\partial b} L'(\psi) f_\psi(\psi) d\psi. \qquad (48)$$

From (43) $L'(\psi) = \sigma_2[b + r(-1 + \Phi(\psi))]$ and $\frac{\partial}{\partial b}L'(\psi) = \sigma_2$. Then Equation (48) simplifies to

$$\frac{\partial^2 v(y_m)}{\partial y_m \partial b} = -1 + \int_{-\infty}^{z^*} f_\psi(\psi) d\psi,$$

which is negative since the second term is a cdf and is always less than 1.

Part 3. This result is implied by *part 1* since there is no change in the local farm's ability to supply the retailer as b changes.

Part 4. Clearly, when the ordering cost from the local farm w_l dropped to $b < w$, the retailer achieves a lower cost than when $w_l = w$, using the policy that is optimal for the latter. Using the optimal policy under backhauling would further reduce the expected one period cost.

∎

A.3 Proof of Proposition 7.

Proof of Proposition 7. At time t, the retailer orders the entire local supply. Thus, at time $t-1$, the retailer's problem is given by

$$\min_{y_m \geq 0} w\,\mathbb{E}_{\epsilon_2,\epsilon_1,\epsilon_s}[(y_m+\mu+\epsilon_s-\lambda-\epsilon_1-\epsilon_2)^+] + (r-w)\,\mathbb{E}_{\epsilon_2,\epsilon_1,\epsilon_s}[(\lambda+\epsilon_1+\epsilon_2-y_m-\mu-\epsilon_s)^+].$$

We define a mean-zero random variable as $\epsilon_1+\epsilon_2-\epsilon_s$ with variance $\sigma_1^2+\sigma_2^2+\sigma_S^2$. Then, the newsvendor solution is $y_m+\mu-\lambda = \sqrt{\sigma_1^2+\sigma_2^2+\sigma_S^2}\Phi^{-1}\big(\frac{r-w}{r}\big)$. ■

B Proofs of the Results in Section 4

Proof of Lemma 2. Consider the formulation (13), and let $0 < q_1 < q_2$. Also let $\mathbf{y}_i = \mathbf{y}(q_i)$ for $i = 1, 2$. By the optimality of $\mathbf{y}_1$ for q_1 (and the feasibility of $\mathbf{y}_2$) we write[5]

$$C(\mathbf{y}_1) - q_1 S(\mathbf{y}_1) < C(\mathbf{y}_2) - q_1 S(\mathbf{y}_2). \tag{49}$$

Similarly, by the optimality of $\mathbf{y}_2$ for q_2 (and the feasibility of $\mathbf{y}_1$), we write $C(\mathbf{y}_1) - q_2 S(\mathbf{y}_1) > C(\mathbf{y}_2) - q_2 S(\mathbf{y}_2)$, which is equivalent to

$$q_2 S(\mathbf{y}_1) - C(\mathbf{y}_1) < q_2 S(\mathbf{y}_2) - C(\mathbf{y}_2). \tag{50}$$

Adding (49) and (50) gives $S(\mathbf{y}_1) < S(\mathbf{y}_2)$. ■

Proof of Proposition 8. Note that $S(\mathbf{y}(q(\rho))) = \rho$. Therefore, it suffices to show that $C(\mathbf{y}(q(\rho))) \leq C(\mathbf{y})$ for all $\mathbf{y}$ with $S(\mathbf{y}) \geq \rho$. Since $\mathbf{y}(q(\rho))$ is optimal for the formulation (13) with penalty $q(\rho)$ we conclude that $C(\mathbf{y}(q(\rho))) - q(\rho)S(\mathbf{y}(q(\rho))) \leq C(\mathbf{y}) - q(\rho)S(\mathbf{y})$ for all $\mathbf{y}$. Then by definition of $q(\rho)$, we write $C(\mathbf{y}(q(\rho))) - q(\rho)\rho \leq C(\mathbf{y}) - q(\rho)S(\mathbf{y})$, which implies $C(\mathbf{y}(q(\rho))) \leq C(\mathbf{y}) - q(\rho)(S(\mathbf{y}) - \rho)$.

Note that $S(\mathbf{y}) - \rho \geq 0$ for all $\mathbf{y}$ with $S(\mathbf{y}) \geq \rho$ (feasible $\mathbf{y}$ for (12)). Therefore, we conclude that for all $\mathbf{y}$ with $S(\mathbf{y}) \geq \rho$, $C(\mathbf{y}(q(\rho))) \leq C(\mathbf{y})$. Thus, the order policy $\mathbf{y}(q(\rho))$ is optimal for the retailer's problem (12). ■

Proof of Proposition 9. The objective function can be written equivalently as follows:

$$w \int_{-\infty}^{y_m+y_l-\lambda-\tilde{\epsilon}_1} (y_m+y_l-\lambda-\tilde{\epsilon}_1-\epsilon_2) f(\epsilon_2) d\epsilon_2 + (r-w) \int_{y_m+y_l-\lambda-\tilde{\epsilon}_1}^{\infty} (\lambda+\tilde{\epsilon}_1+\epsilon_2-y_m-y_l) f(\epsilon_2) d\epsilon_2 \\ - q\Phi\Big(\frac{y_l+y_m-\lambda-\tilde{\epsilon}_1}{\sigma_2}\Big),$$

where f is the pdf of ϵ_2. The first order condition gives the following

$$w \int_{-\infty}^{\infty} f(\epsilon_2) d\epsilon_2 - r \int_{y_m+y_l-\lambda-\tilde{\epsilon}_1}^{\infty} f(\epsilon_2) d\epsilon_2 - \frac{q}{\sigma_2}\phi\Big(\frac{y_l+y_m-\lambda-\tilde{\epsilon}_1}{\sigma_2}\Big) = 0.$$

Rearranging terms give

$$\frac{r-w}{r} = \Phi\Big(\frac{y_l+y_m-\lambda-\tilde{\epsilon}_1}{\sigma_2}\Big) - \frac{q}{\sigma_2 r}\phi\Big(\frac{y_l+y_m-\lambda-\tilde{\epsilon}_1}{\sigma_2}\Big).$$

Note that the right-hand side is strictly increasing in y_l, from which the result follows after truncating y_l to ensure $0 \leq y_l \leq \mu + \tilde{\epsilon}_s$. ■

[5] The strict inequality follows from strict convexity and the uniqueness of the optimal solution.

Proof of Proposition 10. Given the order quantity from the mainstream farm y_m, the retailer's profit under manager's discretion ($y_l^* = \mu + \epsilon_s$) with backhauling is given by

$$(r-b)\lambda - (w-b)\, y_m - b\,\mathbb{E}[\,(y_m + \mu - \lambda + \epsilon_s - \epsilon_1 - \epsilon_2)^+] - (r-b)\,\mathbb{E}[(\lambda - \mu - y_m + \epsilon_1 - \epsilon_2 - \epsilon_s)^+]. \quad (51)$$

Then the retailer's optimal profit under manager's discretion with backhauling subject to the service level, denoted by $\Pi^S_{\text{MB}}(b)$, can be derived from (11) and (51). Therefore, we conclude that $\Pi^S_{\text{MB}}(\cdot)$ is strictly decreasing and continuous in b.

To conclude the proof, let Π^S_{H} and $\Pi^S_{\text{HB}}(b)$ denote the retailer's optimal profit under hybrid sourcing and hybrid sourcing with backhauling policies (under the same service level constraint). First, note that

$$\Pi^S_{\text{HB}}(b) > \Pi^S_{\text{H}} \quad \text{for all } b < w. \quad (52)$$

This follows because backhauling lowers the order cost from the local farm and helps the retailer. Next, we argue that

$$\Pi^S_{\text{MB}}(0) \geq \Pi^S_{\text{HB}}(0) > \Pi^S_{\text{HB}}(b). \quad (53)$$

The second inequality follows because lowering the unit ordering cost b from the local farmer increases the retailer's profit. To see why the first inequality follows, let y_m^*, $y_l^*(\epsilon_1, \epsilon_s)$ denote the optimal policy achieving $\Pi^S_{\text{HB}}(0)$. Then consider the manager's discretion policy with the same y_m, which orders $\mu + \epsilon_s - y_l(\epsilon_1, \epsilon_s)$ more from the local farmer but incurs no additional costs since $b = 0$. Since the optimal manager's discretion policy corresponding to $\Pi^S_{\text{MB}}(0)$ does at least as well as any feasible policy we conclude that $\Pi^S_{\text{MB}}(0) > \Pi^S_{\text{HB}}(0)$.

Combining (52) and (53) gives $\Pi^S_{\text{MB}}(0) \geq \Pi^S_{\text{H}}$. Since $\Pi^S_{\text{MB}}(\cdot)$ is continuous, there exists $\hat{b} > 0$ such that $\Pi^S_{\text{MB}}(b) > \Pi^S_{\text{H}}$ for $b \in (0, \hat{b})$. This proves that for sufficiently low b, the retailer's profit is higher than her profit under hybrid sourcing, and hence also than that under mainstream only ordering policy.

■

C Derivation of Equation (22) in Section 6

Taking the $x(t)$ term out in (21) yields $C_t(x|\pi) = \sum_{t=0}^{\infty} \alpha^t\, \mathbb{E}[wy(t) + L(y(t))] - \sum_{t=0}^{\infty} \alpha^t\, \mathbb{E}[wx(t)|x(t)]$. Manipulating the second term, this can be written as

$$C_t(x|\pi) = \sum_{t=0}^{\infty} \alpha^t\, \mathbb{E}[wy(t) + L(y(t))] - \sum_{t=0}^{\infty} \alpha^{t+1}\, \mathbb{E}[wx(t+1)] - wx(t).$$

Substituting $x(t+1) = [y(t) - D(t)]^+$ and combining the terms on the right-hand side yields

$$C_t(x|\pi) = \sum_{t=0}^{\infty} \alpha^t\, \mathbb{E}[w(y(t) - \alpha[y(t) - \tilde{D}(t)]^+ + L(y(t))] - wx(t).$$

Substituting $L^+(y) = L(y) + wy - \alpha w \mathbb{E}_{\epsilon_2}[(y - \tilde{D})^+]$ yields the statement in (22).