Smart Beta Strategies: the Social Responsibility of Investment Universes Does Matter

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Abstract

In this article we extend the research on smart beta strategies by exploring how using an SRI universe impacts the properties of smart beta portfolios. We focus on four smart beta strategies: the Equally Weighted (EW), the Most Diversified Portfolio (MDP), the Minimum Variance (MV) and the Equal Risk Contribution (ERC). Using different estimators of the matrix of covariances, we apply these strategies to the EuroStoxx universe of stocks, the ASPI and the complement of the ASPI in the EuroStoxx universe from March 15, 2002 to May 1, 2012. We show that smart beta strategies, built on the entire universe, concentrate their solution on non-SRI stocks. Consequently, the portfolios built on the ASPI are more diversified and, tend to have higher turnover. In addition, their tracking error against EuroStoxx, is smaller than those of their respective counterparts built on the two other universes, and their distribution of returns has positive skewness, while those of portfolios built on the two other universes have negative skewness. Consistent with the empirical literature, all the smart beta portfolios built on the ASPI universe in our sample outperform the CW portfolio.

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Socially responsible investment, alternative and smart beta strategies, performance, diversification, turnover, robust covariances matrix

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1 Smart beta strategies and Socially Responsible Investment: Introduction

Against a background of market disappointments, such as poor performance of market capitalization weighted indices and active portfolios, risk-based strategies stand out as financial vehicles for sophisticated institutional investors. Risk-based strategies are heuristic and quantitative asset allocation strategies that are special cases of the risk budgeting allocation approach; the approach itself is one type of alternative weighting approach to asset allocation, the other being the fundamental allocation (Arnott, Hsu, and Moore [2005]). The different alternative weighting strategies are also known as smart beta strategies[1]. These smart beta strategies define the weights of assets in portfolios as functions of individual and common asset risks. The strategies are heuristic because they do not rely on any formal equilibrium model of expected return.

The adoption of smart beta strategies is commonly justified by three principal arguments (Maillard, Roncalli, and Teiletche [2010], Demey, Maillard, and Roncalli [2010]). First, while they implicitly use estimations of expected returns, smart beta strategies do not require any stock return forecasts, which eliminates the challenge of estimating them. This is an advantage compared to Mean-Variance approaches. Second, smart beta strategies aim to improve the risk/return ratio by improving risk diversification. This is an advantage compared to the capitalization-weighted (CW) strategy, which is usually not mean-variance efficient in practice. Third, when back-tested, smart beta strategies outperform the traditional CW investment strategy.

However, smart beta strategies have two drawbacks. Not only there is a lack of theoretical background proving their historical efficiency, but these strategies also involve issues of stability (i.e. turnover) and concentration in terms of weighting of the components of portfolios. To overcome these drawbacks, asset managers follow different implementation approaches. The consequence is that for a given smart beta strategy, institutional investors are faced with the costs involved in choosing from a wide range of implementation approaches[2], which will have major implications for the subsequent characteristics of their portfolios.

In parallel with the rise of smart beta strategies and fuelled by the increasing public concern for sustainable development (Brundtland et al. [1987]), a type of investment generally called socially responsible investment (SRI), is rapidly gaining favour with institutional investors. Briefly, SRI incorporates non-financial criteria into the construction of financial portfolios. These criteria include respecting simple subjective rules (e.g. no investment in gambling or tobacco businesses), or meeting a minimum level of extra-financial performance (e.g. investment in issuers that have low carbon emissions or low rate of fatalities compared to industry competitors). The latter criterion is evaluated by extra-financial rating agencies such as VIGEO.

The popularity of SRI is partly explained by a large literature showing that corporate social performance (CSP) can lead to superior economic and/or financial performance through different mechanisms (Renneboog, Horst, and Zhang [2008], Kitzmueller and Shimshack [2012]). However, a review of these mechanisms is outside the scope of this paper and is covered in a companion paper that focuses on the question of the performance of SRI. The principle of SRI is to incorporate extra-financial criteria into portfolio construction so as to capture these characteristics and yield higher risk-adjusted returns in the long-run[3].

In the light of these two parallel trends, smart beta strategies and SRI, a key question arises for institutional investors. If firms that demonstrate a high level of CSP are different from low CSP firms, are the characteristics of smart beta portfolios modified by an SRI universe? A positive answer would imply that institutional investors should run a new selection process, to decide which implementation approach for a smart beta strategy is in their best interests for the SRI universe. A negative answer would imply that institutional investors could keep their current smart beta portfolio managers and just switch to an SRI universe.

Here, we seek to extend the research on smart beta allocation by examining the impact of using an SRI universe on certain characteristics of smart beta portfolios. We look at four smart beta strategies, the Equally Weighted (EW), the Most Diversified

Portfolio (MDP), which is equivalent to the modified Maximum Sharpe Ratio (MSR) portfolio, the Minimum Variance (MV) and the Equal Risk Contribution (ERC). Using different estimators of the matrix of covariances, we apply these strategies to the EuroStoxx universe of stocks, the ASPI and the complement of the ASPI in the EuroStoxx universe[4].

Six types of impact of using the ASPI universe of stocks emerge from our study. First, smart beta strategies applied on the EuroStoxx favour stocks that do not belong to the ASPI universe. Second, there is increased diversification of the weight and risk measure distributions. Third, smart beta portfolios built on the ASPI universe tend to present higher weight and component turnovers. Fourth, the distributions of returns of portfolios built on the ASPI universe have positive skewness, while with the two other universes, portfolios have distributions of returns with negative skewness. Fifth, the volatility of tracking error against EuroStoxx of smart beta strategies built on the ASPI universe is lower than that of their respective counterparts built on the two other universes. Finally, on the ASPI universe, all the smart beta strategies dominate the CW strategy, which is similar to findings on the two other universes and consistent with the empirical literature.

Thus we are able to conclude that combining the smart beta strategies with the SRI approach does modify some properties of smart beta portfolios. This means that the adoption of SRI is not neutral, and needs particular attention from the institutional investors.

In the rest of the paper we first present the four smart beta strategies examined. Section 2 gives the data and methodology for our back-tests and in section 3 we analyze portfolios characteristics. The two last sections review the robustness of our results regarding risk models and conclude.

2 Smart beta strategies: calculation of weights

According to Demey, Maillard, and Roncalli [2010] there are four common types of smart beta strategies yielding four types of smart beta portfolios. In this section we review these four strategies and their particular risk contribution properties.

The first type is the EW portfolio. The EW portfolio depends solely on the number n of components and its weights w_i are given by:

$$\forall i, w_i = \frac{1}{n} \tag{1}$$

This portfolio is straightforward and presents good out-of-sample performance compared to optimal portfolios (DeMiguel, Garlappi, and Uppal [2009]). It is perfectly diversified in weights, by construction.

The second type is the MV portfolio. The vector of weights w of the MV portfolio, with the variance-covariance matrix Σ, is given by the following optimisation program:

$$\begin{aligned} w &= \arg\min(w'\Sigma w) \\ \text{s.t. } &\sum_i^n w_i = 1 \\ &\forall i, 0 \leq w_i \leq 1 \end{aligned} \tag{2}$$

This portfolio is straightforward to understand: it has the lowest ex ante volatility, does not rely on expected return input and offers good relative performance (Clarke, Silva, and Thorley [2006], Scherer [2011]). In addition, marginal risks (MR) are equal for all the components with a weight different from zero.

$$\forall i, j, (w_i \neq 0 \land w_j \neq 0 \Rightarrow \frac{\delta\sigma(w)}{\delta w_i} = \frac{\delta\sigma(w)}{\delta w_j}) \tag{3}$$

The third type of portfolio, the MDP (Choueifaty and Coignard [2008]) or modified MSR (Martellini [2008]), is more complicated. To obtain the vector of weights w of this portfolio, Choueifaty and Coignard [2008] introduce a diversification measure that is maximized:

$$\begin{aligned} w &= \arg\max(\frac{w'\sigma}{\sqrt{w'\Sigma w}}) \\ \text{s.t. } &\sum_i^n w_i = 1 \\ &\forall i, 0 \leq w_i \leq 1 \end{aligned} \tag{4}$$

This portfolio does not explicitly rely on expected return input (see the introduction of Martellini [2008]); it is more diversified and less sensitive to small modifications in inputs than the MV portfolio. In addition, relative marginal risk (RMR) is equal for all the components with a weight different from zero.

$$\forall i, j, (w_i \neq 0 \wedge w_j \neq 0 \Rightarrow \frac{1}{\sigma_i}\frac{\delta\sigma(w)}{\delta w_i} = \frac{1}{\sigma_j}\frac{\delta\sigma(w)}{\delta w_j}) \tag{5}$$

The last type is the ERC portfolio (Maillard, Roncalli, and Teiletche [2010]), also rather complicated, where the risk contribution (RC) of each asset is the same.

$$\forall i, j, (w_i \frac{\delta\sigma(w)}{\delta w_i} = w_j \frac{\delta\sigma(w)}{\delta w_j}) \tag{6}$$

The composition of this portfolio is given by the following program:

$$\begin{aligned} w &= \arg\min(\sum_{i=1}^{n}\sum_{j=1}^{n}(w_i(\Sigma w)_i - w_j(\Sigma w)_j)^2) \\ &\text{s.t. } \sum_{i}^{n} w_i = 1 \\ &\forall i, 0 \leq w_i \leq 1 \end{aligned} \tag{7}$$

This portfolio does not explicitly rely on expected return input, by construction it is well diversified in terms of weights[5] and risk, and it is less sensitive to slight modifications in inputs than the MV or MDP portfolios (Demey, Maillard, and Roncalli [2010]).

Table 1: Smart beta strategies: a comparison

The table lists the conditions (columns) on stocks necessary for each strategy (lines) to be equivalent either to one other strategy or to the tangent portfolio.

Strategies	Conditions on stocks: Same volatility	Same expected return	Same correlation	Same Sharpe ratio	Equivalent to
EW	X	X	X		Tangent
MV		X			Tangent
MDP/mMSR	X				MV
MDP/mMSR				X	Tangent
ERC			X	X	Tangent
ERC			X		MDP/mMSR
ERC			X ($\rho = \frac{-1}{N-1}$)		MV
ERC	X		X		EW

Table 1, based on the literature, summarizes how the different smart beta strategies stand in relation to each other, and to the tangent portfolio. Note that depending on the statistical properties of the stocks included in the portfolios, different strategies can yield the same allocation and the latter can be the tangent portfolio. In particular, the ERC and the MDP portfolios are to be identical when pairwise correlation is uniform. Since we use the constant correlation matrix of covariances in our analyses, it is important to control for this case.

Finally these different approaches, apart from the EW portfolio, rely on the matrix of variances and covariances Σ. In the next section we present the different risk models we use to estimate our four portfolios.

3 Data and methodology of the study

We run our back-tests using daily returns (adjusted price and arithmetic returns) for three different universes of stocks: the EuroStoxx, the ASPI and the complement of the ASPI in the EuroStoxx universe. We use data from March 15, 2002 to May 1, 2012. Our data sources are, Datastream for prices and composition of the EuroStoxx, and IEM[6] for composition of the ASPI index. We check the reliability of our in-house-built universes by calculating the volatility of the tracking error (TEV) of the CW portfolios with the respective indices. We obtain a TEV of 26.9 bps for the replication of ASPI and a TEV of 11.6 bps for the replication of EuroStoxx, which are common levels of TEV (table 2).

Because the ASPI is a best-in-class index, we can verify that the industrial composition of the different universes are similar (table 3). Finally, we use arithmetic returns and calculate all returns in Euros[7] and, following the indices calculation methodology, we rebalance the portfolios at closing on the third Friday of March, June, September and December. The portfolios weights are allowed to drift between rebalancing dates.

Table 2: Performance of real and replication of CW indices

Performance of real and replication of CW indices from March 15, 2002 to May 1, 2012. The two replicated CW portfolios are benchmarked against their respective real counterparts (i.e. real EuroStoxx and ASPI). We also report statistics on performances and crossed benchmark for the two real indices. Sharpe ratio is calculated against a zero risk free rate. Annualized realized performance is annual rate equivalent to total performance.

	EuroStoxx Replication	**EuroStoxx Real**	**ASPI Replication**	**ASPI Real**
		Historical performance		
Total realized perf. (%)	-26,58	-26,37	-29,74	-29,67
Annualized perf. (%)	-2,99	-2,97	-3,41	-3,40
Volatility (%)	23,14	23,49	24,20	24,52
Sharpe ratio	-0,13	-0,13	-0,14	-0,14
Max. draw down (%)	-61,79	-61,75	-60,30	-60,10
		Performance of tracking		
Daily TE (%)	-0,0004	0,0008	-0,0003	-0,0008
TEV (%)	0,1160	0,2623	0,2699	0,2623
Information ratio	-0,0037	0,0030	-0,0012	-0,0030
Correlation	0,9999	0,9984	0,9999	0,9984

Table 3: Industrial average composition

Column A is Consumer Discretionary, B is Consumer Staples, C is Energy, D is Financials, E is Health Care, F is Industrials, G is Information Technology, H is Materials, I is Telecommunication Services, J is Utilities.

Nb	A	B	C	D	E	F	G	H	I	J	NA	TOTAL
EuroStoxx	41,4	23,2	11,0	63,9	14,1	48,8	14,3	26,1	11,5	18,4	32,8	305,4
Comp	22,9	12,2	7,0	41,3	10,3	30,7	6,0	16,6	6,2	12,5	22,1	187,9
Aspi	18,5	11,0	4,4	23,4	3,6	18,0	9,2	9,5	5,3	5,9	11,0	119,9
%	A	B	C	D	E	F	G	H	I	J	NA	TOTAL
EuroStoxx	13,6%	7,6%	3,6%	20,9%	4,6%	16,0%	4,7%	8,5%	3,8%	6,0%	10,8%	100%
Comp	12,2%	6,5%	3,7%	22,0%	5,5%	16,3%	3,2%	8,8%	3,3%	6,7%	11,8%	100%
Aspi	15,4%	9,2%	3,7%	19,5%	3,0%	15,0%	7,7%	7,9%	4,4%	4,9%	9,2%	100%

For the EuroStoxx and the complement universes of stocks, the weights of CW portfolios are calculated using free float market capitalization based on Datastream information. The EW portfolios weights are given by the number of components, which is around 300 for EuroStoxx and around 180 for the complement of ASPI in the EuroStoxx universe[8]. For MV, MDP, ERC portfolios, we estimate weights by optimizing the respective objective functions introduced in the previous section. For the three optimization programs, constraints are no short-sells and no cash holdings. For the ASPI universe of stocks, the weights of CW portfolios are calculated using information given by IEM. The EW portfolio weights are given by N=120, the number of components of ASPI[9]. For MV, MDP, ERC portfolios, we estimate weights by optimizing the respective objective functions introduced in the previous section. For EuroStoxx and the complement universe, optimization constraints are no short-sells and no cash holdings.

Note that the solutions of the MV, MDP and ERC optimization programs depends on the matrix of variances and covariances (the VCV matrix) of stock returns. The estimation of the VCV matrix is challenging, and consequently the solutions given by the optimizations are not stable, leading to high turnover. To improve the stability of solutions given by MV, MDP and ERC optimizations, different estimators of the VCV matrix have been proposed in the literature, a reminder of the diversity of implementation that investors may face. To control for the possible impact of the VCV matrices, we use four estimators: the empirical, the constant correlation, the shrinkage estimator with the constant correlation VCV matrix, and the shrinkage estimator with the one-factor model VCV matrix (Ledoit and Wolf [2004]). At the outset and at each rebalancing, we update the VCV matrix from a 260-day rolling window of the most recent historical data[10]. Another problem in MV and MDP optimizations is the high concentration of solutions. As examined and proposed by Maillard, Roncalli, and Teiletche [2010], we run MV and MDP optimization programs with upper-bound constraints (5% or 10%) for weights.

As for our method of analysis, we first describe the portfolios that we obtain in terms of number of components, number of differences between portfolios yielded by the same strategy on different universes and, differences in weights for identical components in portfolios yielded by the same strategy applied to different universes. This enables us to compare portfolios. Second we focus on diversification, by reporting for each portfolio and universe, the relative mean difference coefficients for weights, risk budget[11], marginal risk, relative marginal risk and risk contribution. The use of relative mean difference will be explained later. Third we focus on turnover, by reporting for each portfolio and universe, turnover of components and turnover of weights. Regressions are performed for all three steps to analyze the correlation of particular characteristics of portfolios with the strategy and the universe used. These regressions give us the economic and statistical significance of the relations of interest while controlling for particular parameters. Finally, we focus on performance. For each portfolio and universe we report descriptive statistics regarding the statistical properties of the distribution of returns of the different portfolios. We report annualized historical performance, annualized historical volatility, annual Sharpe ratio, historical maximum draw down, correlation with benchmark (i.e. the replications of ASPI or EuroStoxx), mean of daily return, its standard error, their two annualized values, mean daily tracking error[12], volatility of daily tracking error and daily information ratio.

Our default case is the empirical VCV matrix. To develop analyses that are not dependent on the VCV matrix, we also run the regressions on datasets that pool the back tests obtained with the four VCV matrices. We discuss the impact of changing the risk model in section 6.

4 Analysis of portfolios characteristics

Composition and differences in composition of portfolios

We first report and analyze the composition and differences in composition of portfolios (Figures 1, 2), so as to describe the portfolios obtained and to measure degrees of similarity between portfolios yielded by the same strategy applied to the different universes.

First, we analyzed portfolio composition. By simply counting the number of components (Figure 1), we distinguished two types of strategy: strategies that invest in the entire available universe (i.e. CW, EW, ERC) and strategies that pick some stocks from the available universe (i.e. MV, MDP and their bounded versions). Although this typology is obtained with the empirical VCV matrix, it is stable when we switch to other types of VCV. Only the MDP strategy with a constant VCV matrix is modified (cf. Table 1).

Second, we calculated differences in portfolio using two measures of difference. Measure D_1 is the absolute difference in weights w_i between the components of portfolios A and B. With n as overlapping components, this measure is given by the following formula[13]:

$$D_1(A,B) = 1 - \sum_{i}^{n} \min(w_{Ai}, w_{Bi}) \qquad (8)$$

Figure 1: Number of components of portfolios

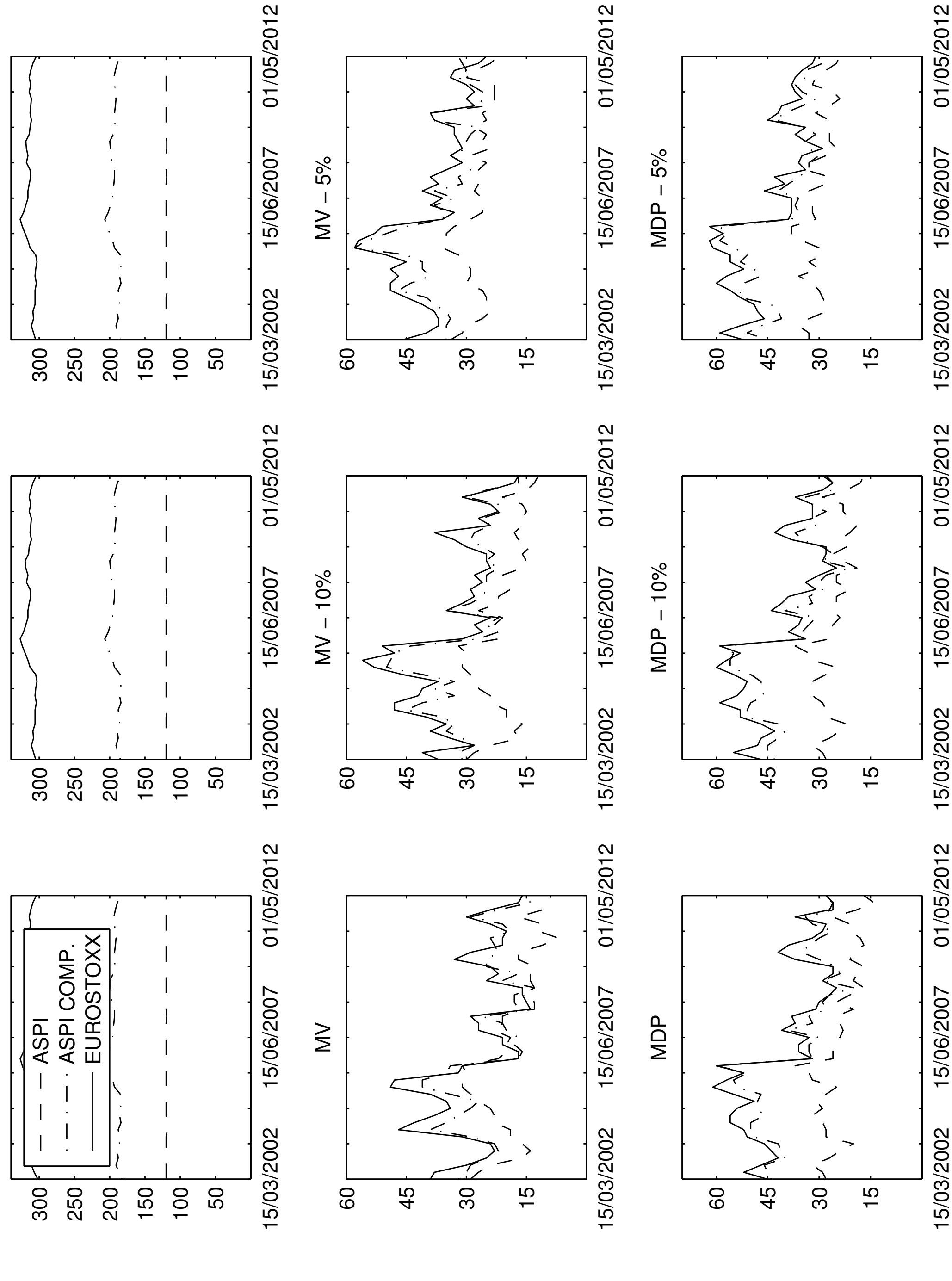


Figure 2: Weights differences

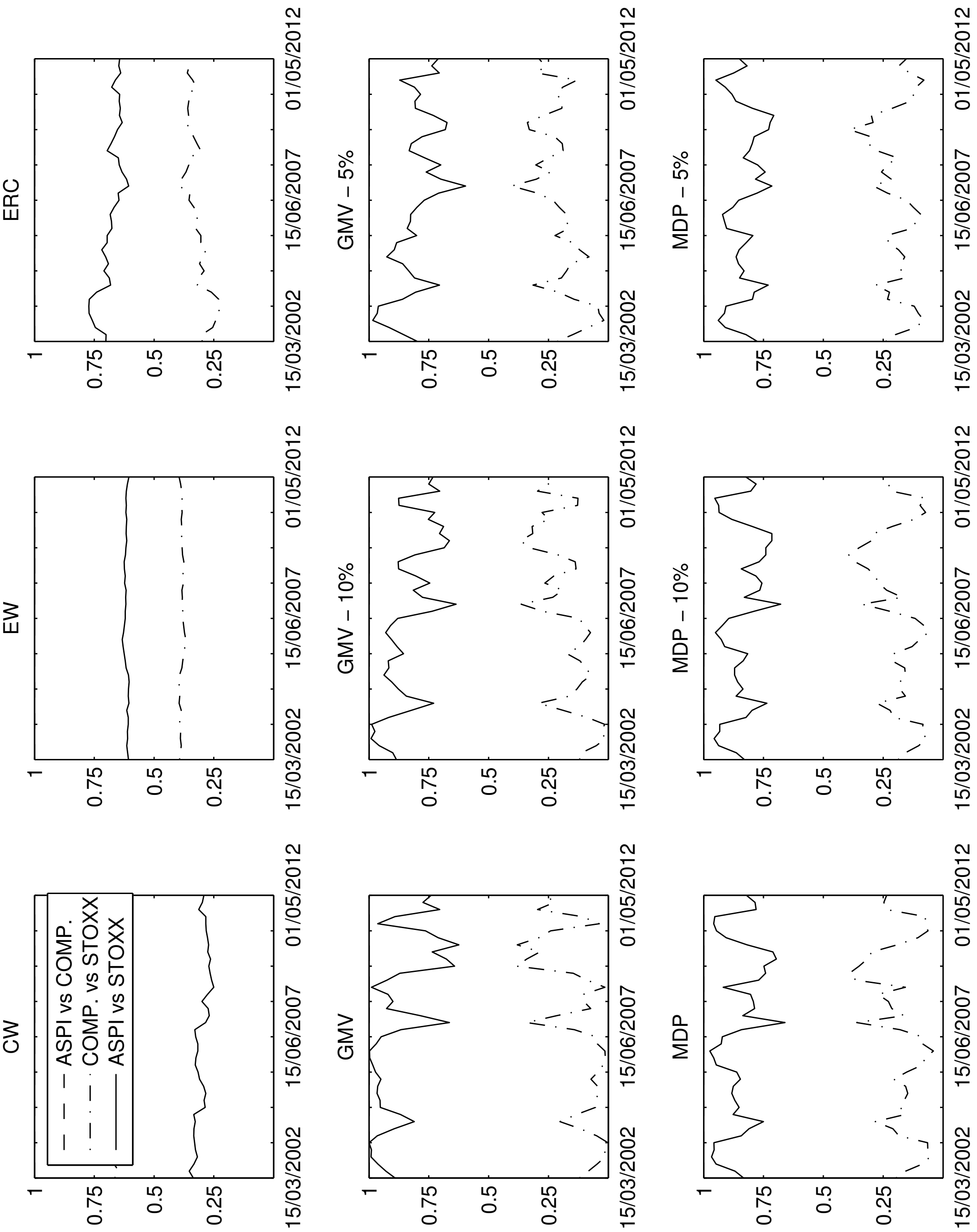


Figure 3: Turnover of weights

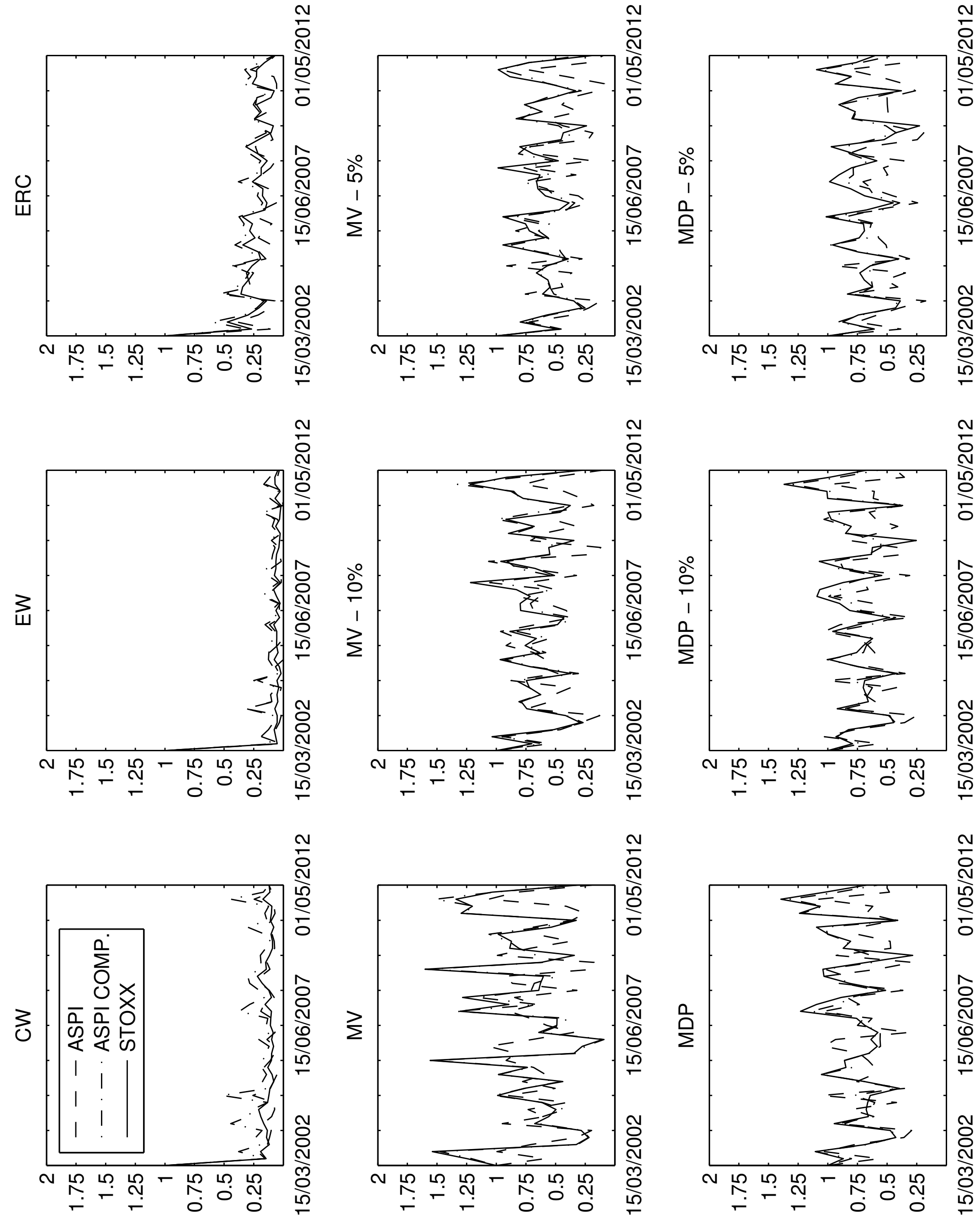
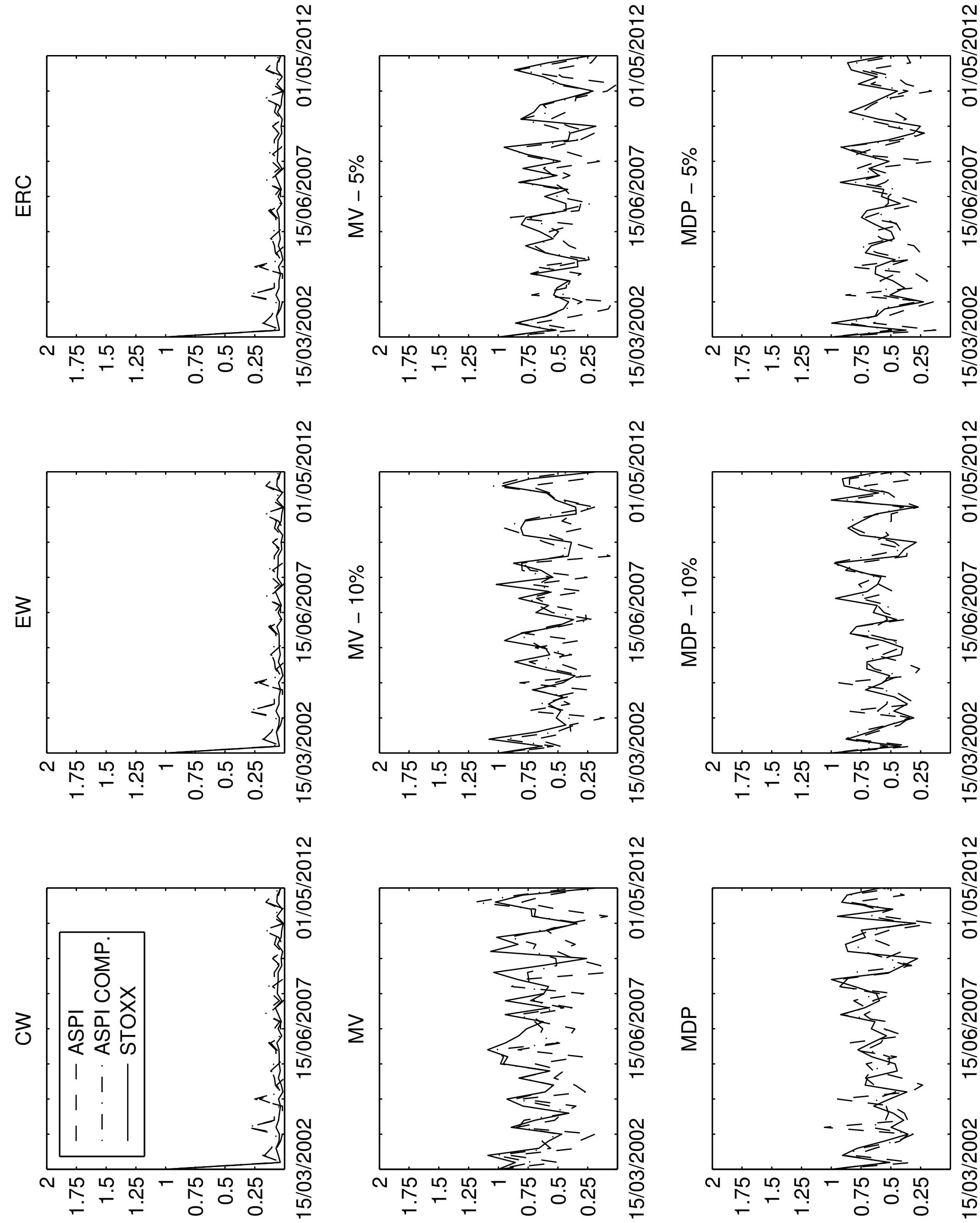


Figure 4: Turnover of components



Measure D_2 is the relative number of differences in the list of components of portfolio A with respect to the list of components of portfolio B. It is given by the following formula[14]:

$$D_2(A,B) = 1 - \frac{\text{card } A \cap B}{\min(\text{card } A, \text{card } B)} \quad (9)$$

Using D_1 in combination with D_2, enables us to allow for the fact that certain strategies only pick some stocks from the available universe. Hence, while the two measures are consistent in the two extreme situations (perfect overlap and perfect difference), they can differ in other situations, especially where there are highly concentrated solutions. Thus, we think it is important to explicitly track differences in lists of components to avoid misleading comparisons based solely on differences in weights.

This second analysis of the overlap of components yields two main findings. First, weight overlapping is much higher with CW than with the other strategies (Figure 2). The CW portfolio built on the ASPI has few differences in weights (i.e. high weight overlapping), while the other strategies built on the ASPI universe have wide differences in weights (i.e. low weight overlapping). Our proposed explanation is the positive correlation between belonging to ASPI and size of firms in our sample. We recall that D_1, our difference in weight, is one minus the sum of the lowest weights of stocks that are in the two portfolios based on the two different universes. As ASPI rules discard about 60% of the EuroStoxx stocks, while we observe only 30% of weight differences, the remaining 40% stocks then must concentrate about 70% of the weights. Consistent with this explanation by size of firms, on average the market values of firms in the ASPI are 3.74 times greater than the market values of firms in EuroStoxx. Finally, the relative mean differences of weights in the CW ASPI we calculate in the next sub-section indicate that firms in the ASPI are in general larger than in the EuroStoxx.

Second, smart beta allocations built on the ASPI universe have very low overlap with portfolios built on the EuroStoxx universe (Figure 2). This means that the optimization programs behind the smart beta allocations concentrate the program solution on firms that are not socially responsible. Hence, on an ex ante basis, portfolios built on the ASPI universe are less optimal than portfolios built on the EuroStoxx and complement of ASPI universes. The latter will be recalled in the section on performance.

Diversification of portfolios

The literature suggests that the advantage of smart beta allocations is better diversification than with the CW allocation. Thus, given that SRI is criticized for reducing opportunities for diversification, our main objective is to analyze how using an SRI universe impacts this strong point of smart beta strategies. We now analyze the diversification of portfolios through diversification of weights and diversification of risk budget, marginal risk, relative marginal risk and risk contribution.

First, we measure diversification of the previously listed characteristics of portfolios. Usually, diversification is measured with the Gini coefficient; however the Gini coefficient is valid only if the support of the analyzed distribution is null or positive. Since some of the characteristics we analyze can take negative values, we measure diversification via relative mean difference[15] (RMD). For a given distribution of measure m, with n observations, we apply the following formula:

$$\text{RMD}_m = \frac{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|m_i - m_j|}{\bar{m}} \quad (10)$$

For each strategy, for each universe and for each VCV matrix, we calculate the RMDs on the entire universe available and at each rebalancing date, for the weight distributions and the four risk measures. We obtain four samples of time series of RMDs that are used in the second step of our diversification analysis.

Second, after analysing these RMD time series we analyze jointly the measures of diversification of the different characteristics of portfolios. Indeed the diversification of a portfolio is a notion that covers different characteristics of the studied portfolio. We pool the five portfolios' characteristics of interest (i.e. weight, risk budget, marginal risk, relative marginal risk and risk contribution) and run regressions of the RMDs on different factors we detail latter. The purpose is to identify, in an unconditional and controlled statistical approach the relationship between diversification and the use of the ASPI universe, while testing for statistical significance. The approach consists in regressing two samples of pooled RMDs of weights and risk measures on universe dummies, strategy dummies, interaction dummies, control dummies and number of components in respective portfolios and universes. Sample A groups RMDs obtained with the empirical VCV. Sample B groups RMDs obtained with the four VCV matrices. The control dummies control for size of portfolio, size of universe of reference, time and other technical controls[16].

The results of the analysis of time series are twofold. First, when we focus on the degree of diversification of strategies built on the same universe, we observe rankings similar to Maillard, Roncalli, and Teiletche [2010]. The most diversified are the EW and the ERC, followed by the CW, and finally the MDP and the MV, the most concentrated in risk and weights. Second, when we focus on the degree of diversification of strategies over the three universes and the five measures, we observe no modification in ranking when switching from ASPI to EuroStoxx or to the complement of the ASPI in the EuroStoxx universe of stocks. However, it emerges that portfolios constructed on the ASPI universe tend to be the most diversified.

Table 4: Analysis of diversification

$$\text{RMD}_{it} = \beta_0 + \beta_1 * \text{D}_i^{ASPI} + \beta_2 * \text{D}_i^{A\bar{S}PI} + \beta_3 * \text{D}_i^{ERC} + \beta_4 * \text{D}_i^{EW} + \beta_5 * \text{D}_i^{MDP} + \beta_6 * \text{D}_i^{MV} + \beta_7 * \text{D}_i^{ASPI} * \text{D}_i^{Strategies} + \beta_8 * \text{D}_i^{A\bar{S}PI} * \text{D}_i^{Strategies} + \beta_9 * \text{Controls}_{it} + \varepsilon_{it}$$

We regress on group dummies measure of concentration of the distributions of the different characteristics of interest. These regressions are estimated with a FGLS estimator with HC p-values Beck and Katz [1995]. Sample A is a panel of 135 series with 41 dates that groups RMDs obtained with the empirical VCV. Sample B is a panel of 450 series with 41 dates that groups RMDs obtained with the four VCV matrices. Universe size and Portfolio size are actual size divided by 100. We control for the type of measure and for the cases of perfect diversification that are predicted by theory. These regressions show significant positive correlation between diversification and use of the ASPI universe. Significant coefficients at the confidence level of 10% and below are in bold.

	Sample A			Sample B		
	coef.	**s.d.**	**p-value**	**coef.**	**s.d.**	**p-value**
cst	**2,02**	**0,30**	*0,00*	**1,99**	**0,34**	*0,00*
ASPI	**-0,40**	**0,18**	*0,03*	**-0,41**	**0,20**	*0,04*
A$\bar{S}$PI	-0,29	0,27	*0,28*	-0,30	0,35	*0,38*
ERC	-0,41	0,32	*0,20*	-0,46	0,29	*0,12*
EW	**-0,41**	**0,22**	*0,07*	-0,42	0,30	*0,16*
MDP	-0,28	0,24	*0,24*	-0,39	0,26	*0,14*
MV	-0,23	0,22	*0,29*	-0,31	0,26	*0,22*
ASPI*ERC	0,03	0,21	*0,89*	0,05	0,19	*0,81*
ASPI*EW	0,06	0,12	*0,60*	0,06	0,15	*0,68*
ASPI*MDP	**0,35**	**0,21**	*0,09*	0,30	0,22	*0,18*
ASPI*MV	**0,34**	**0,19**	*0,08*	**0,39**	**0,21**	*0,07*
A$\bar{S}$PI*ERC	0,10	0,31	*0,73*	0,10	0,34	*0,77*
A$\bar{S}$PI*EW	0,11	0,26	*0,67*	0,11	0,34	*0,75*
A$\bar{S}$PI*MDP	0,28	0,28	*0,31*	0,25	0,35	*0,48*
A$\bar{S}$PI*MV	0,30	0,28	*0,28*	0,32	0,34	*0,35*
Port. size	**-0,20**	**0,02**	*0,00*	**-0,22**	**0,02**	*0,00*
Univ. size	0,04	0,06	*0,54*	0,05	0,05	*0,34*
Time	-0,00	0,00	*0,25*	-0,00	0,00	*0,47*
Bound 5%	-0,05	0,11	*0,66*	-0,05	0,09	*0,59*
Bound 10%	-0,02	0,09	*0,86*	-0,01	0,08	*0,86*
Controls		Yes			Yes	
Adj. R squared		0,29			0,28	

The results of the controlled regressions (table 4) confirm that the ASPI universe is correlated with higher diversification, how strongly depending on the strategy used. With the MV and the MDP, the effect on diversification is weaker than with the CW, the EW, the ERC, whatever the VCV matrix used. The regressions also confirm that the four smart beta strategies yield more diversified distributions than the CW strategy, and that the EW and ERC strategies are the most strongly correlated with higher diversification. Two further observations emerge from the regressions: first, adding bounds to the MV and MDP strategies improves diversification but this improvement is not statistically significant; second, the complement of the ASPI universe is also correlated with more diversified distributions; once again, however this is not statistically significant. Third, portfolio size is positively related to diversification of distributions: the larger the portfolio, the more diversified it is.

Turnover of portfolios

The literature identifies one drawback of smart beta allocations as being a higher level of turnover than with CW allocation, leading to higher transaction costs. Here, therefore, we examine how using an SRI universe impacts this disadvantage of smart beta allocations, in two steps.

First, we calculate two measures of the turnover of portfolios. T_1 measures the turnover of the weights, and is defined by the following formula[17] (Demey, Maillard, and Roncalli [2010]):

$$T_1(t) = \sum_{i}^{n} |w_i(t) - w_i(t-1)| \qquad (11)$$

T_2 measure the turnover of components at a rebalancing date. For a portfolio that contains set of stocks A_t at time t, with IN_t the set of entering components at time t and OUT_t the set of exiting components at time t, component turnover is given by the following formula[18]:

$$T_2(t) = \frac{\text{card } IN_t}{\text{card } A_t} + \frac{\text{card } OUT_t}{\text{card } A_{t-1}} \qquad (12)$$

Using measure of turnover T_1 with measure T_2 enables us to allow for the fact that some strategies only pick some stocks out of the available universe. Since there is a difference between handling concentrated turnover and handling a diversified turnover, we think it is important to explicitly keep track of the number of components that change at rebalancing date. We calculate the two measures of turnover at each rebalancing date, for each strategy, for each universe and for each VCV matrix. In total, we obtain 8 samples of time series of measures of turnover, which are used in the second step of our turnover analysis.

Second, after a basic analysis of these times series, we run regressions of each measure of turnover on different factors. As with diversification, the aim is to have controlled statistical measurements of the relationship between turnover and use of the ASPI universe while testing for statistical significance. The approach consists in regressing two samples of measure T_1 and two samples of measure T_2 against universe dummies, strategy dummies and number of components in portfolios and universes. For each measure, sample A groups the measures of turnover obtained with the empirical VCV, while sample B groups the measures of turnover obtained with the four VCV matrices.

Our basic analysis reveals that the weight turnover is the lowest for EW portfolios (Figure 3), the second lowest turnover being for the CW strategy. The CW strategy is not the one with the lowest turnover in our case, contrary to Carvalho, Lu, and Moulin [2012]. The third lowest turnover is for the ERC strategy. MV and MDP portfolios have similar weight turnovers, those of the MV portfolios, however, being more volatile than the turnovers of the MDP portfolios. Similarly, component turnover is the lowest for the EW, the CW and the ERC strategies (Figure 4). However, when measure T_2 is used, the three strategies have the same component turnover. Since they invest in the entire available universe, this component turnover requests solely from universe modifications. MV and MDP strategies have similar component turnovers; however the turnover of the MV strategy is more volatile than that of the MDP strategy. No modification in these results is observed when switching from ASPI to EuroStoxx or to the complement of the ASPI in the EuroStoxx universe of stocks. And overall we cannot tell whether the utilisation of ASPI leads to higher turnover or not by simply looking at these time series.

The results of the controlled regressions show that the utilization of the ASPI universe is associated to a larger turnover than the complement of ASPI and the EuroStoxx universe, but this relationship is not statistically significant[19].

Performance of portfolios

Table 5 reports annual performance, annual Sharpe ratio, historical maximum draw down, annualized mean of daily return, historical volatility, skewness and kurtosis of daily returns for all the strategies, and for the three universes.

In Table 6, we report the correlation with the respective universe benchmark (i.e. the replications of ASPI or EuroStoxx), the average daily tracking error[20], the volatility of that daily tracking error and the daily information ratio. Finally, in Table 7 we report the correlation of all the strategies on the three universes with the replication of EuroStoxx, the average daily tracking error[21], the volatility of that daily tracking error and the daily information ratio[22].

We recall that strategies investing in the entire available universe (i.e. CW, EW, ERC) can be distinguished from those that pick some stocks out of the available universe (i.e. MV, MDP and their bounded versions). When all the strategies are compared (Table 5), all except the unbounded MV strategy outperform the CW portfolio in the three universes. This is in line with the literature.

When the strategies picking only some stocks out of the available universe are compared, the MV and the MDP portfolios are seen to be taking big bets. The results of our back tests illustrate what happens when one of these bets is "lost" or "won". For instance, at the end of February 2009, Petroleos (CEPSA) lost 57% of its value in three days. At that time, using the EuroStoxx universe, the MV strategy had more that 80% invested in that stock and the MDP strategy had about 53% invested. Conversely, the performance of the MV strategy applied to the ASPI universe of stocks illustrates what happens when the bets are "won". Because of this manner of weighting, the MV and MDP strategies have the highest kurtosis and skewness of all the strategies. Finally, of all the strategies, the MV and the MDP have the lowest ex post volatility and the highest TEV. Their constrained versions have lower ex post volatility and lower TEV. Like Carvalho, Lu, and Moulin [2012], we observe that the constrained versions of the MV and MDP strategies show economically significant higher Sharpe ratios than the other strategies (Tables 5, 6).

When strategies investing in the entire available universe of stocks are compared to stock picking strategies, we observe that they show distributions of returns that are less asymmetric and prone to extremes. Moreover, of the strategies investing in the entire available universe of stocks, it is the ERC strategy that has the lowest ex-post volatility, the smallest maximum drawdown, the highest TEV and the highest Sharpe ratio. The ERC strategy also has the highest return and the lowest ex-post volatility (Tables 5, 6).

We now turn to the impact of the universe of stocks on the characteristics of a given type of strategy. When we analyze the impact of the ASPI universe on the performance of strategies, the unbounded MV and MDP strategies on the ASPI universe outperform all the other strategies on all the other universes (Table 5). In Sharpe ratio, the unbounded MV outperforms the other strategies, and the smart beta strategies built on the ASPI universe generally outperform their counterparts built on the two other universes. The information ratios obtained with their respective benchmarks and those obtained with the replication of the EuroStoxx (Tables 6, 7) show similar results, with the exception that unbounded MV no longer outperforms the others. In all cases, the unbounded MV and MDP strategies on the ASPI universe, yield better mean-variance portfolios with high kurtosis and positive skewness. The latter is also true for the strategies that invest in the entire universe (i.e. CW, EW, ERC). However, despite their positive skewness, the CW, EW and ERC strategies yield significantly poorer mean-variance portfolios. Finally, the smart beta strategies on the ASPI, except for the 1/n, are seen to have lower volatility of tracking error against the EuroStoxx than any of the smart beta strategies applied on the two other universes (Table 7).

When the complement of ASPI universe is used, the strategies investing in the entire universe yield the best performing portfolios, with highest returns and lowest ex-post volatility (Table 5). However the distribution of returns of the CW, EW and ERC portfolios built on the complement of ASPI tends to be exposed to negative extreme returns. This trend gets stronger when we switch to strategies that pick some stocks out of the available universe.

Table 5: Statistics of performance for the three universes

In this table we report statistics of performance of all smart beta and CW strategies that are simulated on the respective universe from March 15, 2002 to May 1, 2012. According to annualized performance, the two best strategies are the MV and MDP smart beta strategies applied to the ASPI universe. According to Sharpe ratio the two best strategies, excluding the constrained ones, are also the MV and MDP smart beta strategies applied to the ASPI universe. Note the positive skewness of portfolios built on the ASPI universe. The Sharpe ratio is calculated against a zero risk free rate. Annualized expected return is expected daily return times 260. Volatility is standard deviation of daily return times $\sqrt{260}$. Annualized realized performance is annual rate equivalent to total performance.

		1/n	**ERC**	**MV**	**MV - 10%**	**MV - 5%**	**MDP**	**MDP - 10%**	**MDP - 5%**	**CW**
EuroStoxx	Annualized perf. (%)	0,46	1,52	-3,43	4,51	4,73	0,97	3,75	2,72	-2,99
	Annual Sharpe ratio	0,02	0,09	-0,22	0,37	0,41	0,06	0,25	0,20	-0,13
	Max. draw down (%)	-63,93	-61,28	-72,97	-46,84	-43,38	-60,68	-48,52	-49,58	-61,79
	Annualized expected return (%)	2,72	2,98	-2,18	5,14	5,29	2,20	4,81	3,56	-0,36
	Volatility (%)	21,24	17,19	15,73	12,18	11,59	15,77	15,13	13,29	23,14
	Skewness daily return	-0,06	-0,15	-7,51	3,15	1,53	1,41	4,09	0,87	0,12
	Kurtosis daily return	8,12	9,35	155,16	78,80	39,80	102,14	120,43	26,59	8,39
ASPI	Annualized perf. (%)	-0,89	0,77	7,42	6,15	3,86	4,79	4,62	3,95	-3,41
	Annual Sharpe ratio	-0,04	0,04	0,50	0,41	0,27	0,24	0,27	0,25	-0,14
	Max. draw down (%)	-64,63	-59,82	-42,44	-44,49	-49,79	-50,41	-53,35	-52,76	-60,30
	Annualized expected return (%)	1,79	2,75	8,24	7,08	4,83	6,64	5,96	5,12	-0,55
	Volatility (%)	23,19	19,91	14,85	15,11	14,45	20,29	17,13	15,84	24,20
	Skewness daily return	0,04	0,05	3,57	3,64	0,72	7,11	2,76	0,36	0,18
	Kurtosis daily return	7,82	8,95	94,17	94,51	22,23	236,07	74,64	16,63	8,24
COMP. ASPI	Annualized perf. (%)	1,27	1,91	-4,30	3,28	3,53	0,22	2,68	1,90	-1,97
	Annual Sharpe ratio	0,06	0,12	-0,27	0,29	0,31	0,02	0,21	0,14	-0,09
	Max. draw down (%)	-63,65	-61,84	-74,29	-50,63	-46,99	-61,16	-48,75	-50,57	-65,69
	Annualized expected return (%)	3,32	3,24	-3,05	3,87	4,13	1,15	3,46	2,75	0,31
	Volatility (%)	20,30	16,39	15,93	11,26	11,43	13,57	12,79	13,13	21,44
	Skewness daily return	-0,12	-0,22	-7,43	0,03	-0,32	-2,69	-0,28	-0,36	-0,04
	Kurtosis daily return	8,37	9,64	149,63	11,13	10,94	41,46	10,82	11,08	8,40

Table 6: Statistics for benchmarks against respective CW portfolio for the three universes

All strategies are benchmarked against the respective in-house CW portfolio from March 15, 2002 ti May 1, 2012. According to information ratio, the best strategy is the 1/n applied on the EuroStoxx and complementary of ASPI universes, followed by the MV, the ERC and the MDP strategies built on the ASPI universe.

		1/n	**ERC**	**MV**	**MV - 10%**	**MV - 5%**	**MDP**	**MDP - 10%**	**MDP - 5%**	**vs CW**
EuroStoxx	Daily TE (%)	0,0118	0,0129	-0,0070	0,0212	0,0217	0,0099	0,0199	0,0151	-
	TEV (%)	0,3443	0,5491	1,3786	1,1413	1,0363	1,2246	1,1347	0,9740	-
	Information ratio	0,0344	0,0235	-0,0051	0,0186	0,0210	0,0081	0,0175	0,0155	-
	Correlation	0,95	0,96	0,81	0,85	0,81	0,93	0,89	0,92	-
ASPI	Daily TE (%)	0,0090	0,0127	0,0338	0,0294	0,0207	0,0276	0,0250	0,0218	-
	TEV (%)	0,3246	0,4415	1,1338	1,0801	0,8884	1,2798	1,0286	0,8486	-
	Information ratio	0,0277	0,0287	0,0298	0,0272	0,0233	0,0216	0,0243	0,0257	-
	Correlation	0,95	0,92	0,66	0,73	0,84	0,68	0,74	0,76	-
COMP. ASPI	Daily TE (%)	0,0116	0,0113	-0,0129	0,0137	0,0147	0,0032	0,0121	0,0094	-
	Standard dev. daily TE (%)	0,2619	0,4638	1,2770	0,9362	0,8510	0,9656	0,8320	0,7786	-
	Information ratio	0,0443	0,0243	-0,0101	0,0146	0,0173	0,0033	0,0146	0,0120	-
	Correlation	0,97	0,97	0,80	0,93	0,91	0,94	0,95	0,96	-

Table 7: Statistics for benchmarks against EuroStoxx for the three universes

All strategies are benchmarked against the in-house EuroStoxx CW portfolio from March 15, 2002 to May 1, 2012. According to the information ratio, the best strategies are the 1/n, applied to the three universes, then the ERC and MV built on the ASPI universe. The volatility of tracking error of the 1/n, ERC and MV smart beta strategies applied on the ASPI universe is smaller than the volatility of tracking error of the same strategies on the other universes.

		1/n	**ERC**	**MV**	**MV - 10%**	**MV - 5%**	**MDP**	**MDP - 10%**	**MDP - 5%**	**vs CW**
EuroStoxx	Daily TE (%)	0,0118	0,0129	-0,0070	0,0212	0,0217	0,0099	0,0199	0,0151	-
	Standard dev. daily TE (%)	0,3443	0,5491	1,3786	1,1413	1,0363	1,2246	1,1347	0,9740	-
	Information ratio	0,0344	0,0235	-0,0051	0,0186	0,0210	0,0081	0,0175	0,0155	-
	Correlation	0,95	0,96	0,81	0,85	0,81	0,93	0,89	0,92	-
ASPI	Daily TE (%)	0,0083	0,0120	0,0331	0,0286	0,0200	0,0269	0,0243	0,0211	-0,0007
	Standard dev. daily TE (%)	0,2618	0,3733	1,0909	1,0376	0,8314	1,2572	0,9870	0,7881	0,1398
	Information ratio	0,0317	0,0321	0,0303	0,0276	0,0240	0,0214	0,0247	0,0268	-0,0050
	Correlation	0,95	0,93	0,68	0,75	0,85	0,69	0,75	0,77	1,00
COMP. ASPI	Daily TE (%)	0,0142	0,0139	-0,0103	0,0163	0,0173	0,0058	0,0147	0,0120	0,0026
	Standard dev. daily TE (%)	0,4559	0,6403	1,3806	1,0551	0,9768	1,0922	0,9678	0,9202	0,3201
	Information ratio	0,0311	0,0216	-0,0075	0,0154	0,0177	0,0053	0,0152	0,0130	0,0081
	Correlation	0,95	0,96	0,79	0,91	0,89	0,93	0,93	0,95	0,99

Hence, the distribution of returns of the MV and MDP portfolios built on the complement of ASPI have high kurtosis and negative skewness. This is consistent with the observation that investors perceive a correlation between extreme specific risk and weak social performance (Waddock and Graves [1997], Hong and Kacperczyk [2009]), and with empirical findings (Boutin-Dufresne and Savaria [2004]).

When the EuroStoxx universe is used, we obtain statistics that are similar to these obtained with the complement of ASPI. This is consistent with the high level of overlapping previously revealed. The MV portfolios built on the EuroStoxx and complement of ASPI universes also have ex-post volatilities higher than the volatility of the ASPI MV portfolio. This observation, together with that on the optimality of smart beta solutions (cf. page 11), illustrates the gap between ex ante optimisation and ex post realisation. It may also illustrate the lower quality of the statistical inputs obtained with the EuroStoxx and complement of ASPI universes.

5 Robustness

As previously introduced, to treat the issue of stability of solutions given by MV, MDP and ERC optimizations we used four different estimations of the VCV matrix: the empirical, the constant correlation and two shrinkage estimators[23] (Ledoit and Wolf [2004]). The different analysis we report in this paper are done with the empirical VCV matrix sample, and with the sample pooling the four different VCV matrices.

Whatever the estimator, using an SRI universe is seen to impact the characteristics (i.e. diversification) of smart beta portfolios to the same degree. However, we find some differences in the degree to which use of smart beta strategies affects the performance of SRI portfolios. For example, using the Sharpe ratio, non-reported regression shows that the constant VCV matrix yields portfolios with significantly poorer performance. We observe lower returns and higher variance of returns than for portfolios built with other VCV matrices. In addition, the shrinkage estimators of the VCV matrix yield portfolios with better performance than portfolios built with empirical estimators of the VCV matrix; but this latter observation is not statistically significant.

However, the use of more sophisticated estimators of the VCV matrix leads to other significant advantages. Non-reported regressions show that more sophisticated VCV matrices significantly decrease the turnover of weights and components. The smallest improvement is obtained with the shrinkage toward the constant VCV matrix. The shrinkage toward the one-factor model and the constant VCV matrix are equivalent.

6 Conclusion

Our intention here was to further explore smart beta allocation by examining how using an SRI universe impacts the characteristics of smart beta portfolios. We studied four smart beta strategies, the EW, the MDP, the MV and the ERC, using three universes of stocks, the EuroStoxx, the ASPI and the complement of ASPI universe. We worked with four different estimators of the VCV matrix: the empirical, the constant, the matrices shrunk towards a constant and towards a one-actor model.

Six types of impact of using the ASPI universe of stocks emerge from our study. First, smart beta strategies applied on the EuroStoxx favour stocks that do not belong to the ASPI universe. In fact, the lists of components and the weights of overlapping components in EuroStoxx and ASPI differ widely. Second, there is increased diversification of the weight and risk measure distributions. Diversification also increases when we use the complement of the ASPI universe; however the correlation is not statistically significant and is lower than the one obtained with the ASPI universe. These observations do not depend on type of VCV. Third, smart beta portfolios built on the ASPI universe tend to present higher weight and component turnovers. Again, these observations do not depend on type of VCV. Fourth, the distributions of returns of portfolios built on the ASPI universe have positive skewness, while with the two other universes, portfolios have distributions of returns with negative skewness. Fifth, the volatility of tracking error against EuroStoxx of smart beta strategies built on the

ASPI universe is lower than that of their respective counterparts built on the two other universes. Moreover, on the ASPI universe, all the smart beta strategies dominate the CW strategy, which is similar to findings on the two other universes and consistent with the empirical literature.

Hence, while recalling the usual limitations of backtesting, we conclude that using smart beta strategies in combination with the SRI approach somewhat modifies the properties of smart beta portfolios. Adopting SRI thus cannot be considered neutral and warrants careful attention from the institutional investor. A valuable extension of this work would be to check the robustness of our results using a different SRI universe with different rating methodology and covering different geographical zones.

Notes

[1]Though we are interested in risk-based alternative weighting, we will stick to the term "smart beta" in the rest of the article.

[2]Some of the implementation choices will be discussed in this paper, in the section on data and methodology.

[3]The second justification for adoption of SRI ressembles the intuition justifying alternative weighting schemes. Actually, SRI can be considered a smart beta approach, between fundamental and risk based allocations, where assets that do not match extra-financial criteria are given a weight equal to zero. Fundamental allocations define the weights as a function of issuers' fundamental statistics. See Arnott, Hsu, and Moore [2005].

[4]The EuroStoxx is a subset of the EuroStoxx 600 that contains a variable number of stocks, roughly 300, traded in Eurozone countries. The ASPI is a subset of EuroStoxx that contains the 120 best rated stocks. This social performance rating is given by VIGEO. The complement of the ASPI in the EuroStoxx universe is the universe of about 180 stocks that are in the EuroStoxx but not in the ASPI.

[5]There is no weight equal to zero in the original theory, but in practice see the numerical approach of Carvalho, Lu, and Moulin [2012], and the analytical work of Clarke, Silva, and Thorley [2012] that shows why stocks with particular negative values of the beta with an ERC portfolio can be excluded from the ERC.

[6]IEM is the firm in charge of calculation methodology for the ASPI. VIGEO is a provider of social performance ratings and sponsor of the ASPI.

[7]By construction EuroStoxx is a Euro Zone universe.

[8]As previously stated, the EuroStoxx is a subset of the EuroStoxx 600 that contains a variable number of stocks, roughly 300.

[9]For 2 rebalancing dates ASPI is defined by N=118 and N=119.

[10]For some stocks historical series are shorter than the VCV estimation window. For the ASPI universe, this concerns two stocks out of 238, the smallest window is 100 days. For EuroStoxx and complement of ASPI universe, this concerns 53 stocks out of 536, the smallest windows is 12 days.

[11]Risk budget is defined as the product of the weight of component i combined with its volatility.

[12]The benchmarks used are our replications of ASPI and EuroStoxx CW indices.

[13]When D_1 equals 1, it means that the two portfolios do not overlap. The portfolios have different lists of components. When D_1 equals 0, it means that the two portfolio are identical.

[14]When D_2 equals 1, it means that the two lists of components do not intersect. When D_2 equals 0 it means that one list is equal to, or included in, the other.

[15]The RMD is closely related to the Gini coefficient. The closer the relative mean difference gets to zero the less concentrated the distribution is.

[16]We control for the case of perfect diversification for the different VCV matrices. That is, the EW and weights, the MDP, ERC and the risk contribution. We control for the different types of characteristics analyzed.

[17]By definition T_1 is between 0 and 2 for one rebalancing and, for the first rebalancing, the turnover equals 1.

[18]By definition T_2 is between 0 and 2 for one rebalancing and, for the first rebalancing, the turnover equals 1.

[19]We do not report the results because of space constraints. They are available from the authors upon request

[20]The benchmarks used are our replications of ASPI and EuroStoxx CW indices.

[21]The benchmarks used are our replications EuroStoxx CW indices.

[22]The results in these three tables are obtained with empirical covariance matrices, using daily returns for three different universes of stocks, the EuroStoxx, the ASPI and the complement of the ASPI in the EuroStoxx universe.

[23]Shrinkage targets are the constant correlation and the one-factor market model VCV matrices.

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